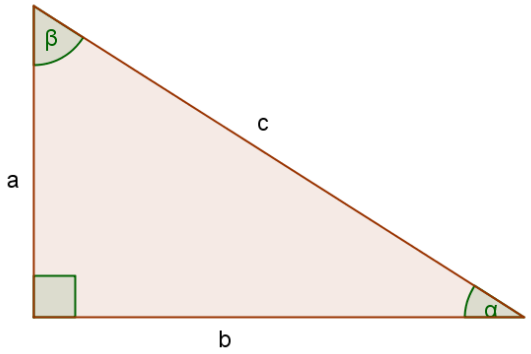
Ponavljanje za pismeni ispit – trigonometrija pravokutnog trokuta

1. Izračunaj kutove, opseg i površinu pravokutnog trokuta u kojem je duljina jedne katete b = 34, a hipotenuze c = 50.



$\cos\alpha = \frac{b}{c} = \frac{34}{50}$

$\boxed{\alpha = 47°9'23''}$

$\beta = 90° - \alpha = 89°59'60'' - 47°9'23''$

$\boxed{\beta = 42°50'37''}$

$a^2 = c^2 - b^2$

$a = \sqrt{c^2 - b^2} = \sqrt{50^2 - 34^2} = \sqrt{2500 - 1156} = \sqrt{1344}$

$\boxed{a = 36.66}$

$O = a + b + c = 36.66 + 34 + 50$

$\boxed{O = 120.66}$

$P = \frac{a \cdot b}{2} = \frac{36.66 \cdot 34}{2}$

$\boxed{P = 628.32}$

2. Odredi mjeru većeg šiljastog kuta pravokutnog trokuta ako je omjer duljina kateta $5 : 7$.

ako uzmemo da je $a : b = 5 : 7$, tražiti ćemo kut β, koji se nalazi nasuprot dulje stranice b

$tg\beta = \frac{b}{a}$

$tg\beta = \frac{7}{5}$

$\boxed{\beta} = tg^{-1}\left(\frac{7}{5}\right) = \boxed{54°27''44''}$

3. U pravokutnom trokutu je mjera jednog šiljastog kuta 7 puta veća od mjere drugog šiljatog kuta. Ako je dulja kateta duga 15 cm, odredi opseg i površinu tog trokuta.

ako uzmemo da je $\alpha = 7 \cdot \beta$, dulja kateta će biti $a = 15$ cm

$\alpha + \beta = 90°$

$7\beta + \beta = 90°$

$8\beta = 90° \quad / : 8$

$\beta = 11°15''$

$tg\beta = \frac{b}{a}$

$b = a \cdot tg\beta = 15 \cdot tg11°15' = 2.984$ cm

$c^2 = a^2 + b^2$

$c = \sqrt{a^2 + b^2} = \sqrt{15^2 + 2.984^2} = 15.294 \text{ cm}$

$\boxed{O} = a + b + c = 15 + 2.984 + 15.294 = \boxed{33.27 \text{ cm}}$

$\boxed{P} = \frac{a \cdot b}{2} = \frac{15 \cdot 2.984}{2} = \boxed{22.38 \text{ cm}^2}$

4. Odredi nepoznate elemente pravokutnog trokuta ako je $c - b = 2$ i $\alpha = 67°22''48''$.

$\alpha + \beta = 90°$

$\boxed{\beta} = 90° - \alpha = 90° - 67°22''48'' = \boxed{22°37''12''}$

$\cos\alpha = \frac{b}{c}$

$b = c \cdot \cos\alpha$

$b = c \cdot \cos 67°22'48''$

$b = 0.385c$

$c - b = 2$

$c - 0.385c = 2$

$0.615c = 2 \quad / : 0.615$

$\boxed{c = 3.252}$

$\boxed{b} = 0.385c = \boxed{1.252}$

$a^2 = c^2 - b^2$

$\boxed{a} = \sqrt{c^2 - b^2} = \sqrt{3.252^2 - 1.252^2} = \boxed{3.001}$

5. Odredi nepoznate elemente pravokutnog trokuta ako je zadana površina $P = 180 \text{ cm}^2$ i kut $\alpha = 43°36''10''$.

$\alpha + \beta = 90°$

$\boxed{\beta} = 90° - \alpha = 90° - 43°36''10'' = \boxed{46°23''50''}$

$P = \frac{a \cdot b}{2}$

$a \cdot b = 2P$

$a \cdot b = 360 \qquad (*)$

$\text{tg}\alpha = \frac{a}{b}$

$a = b \cdot \text{tg}\alpha$

$a = b \cdot \text{tg}43°36'10''$

$a = 0.952b$

povratak u (*)

$0.952b \cdot b = 360$

$0.952b^2 = 360 \quad / : 0.952$

$b^2 = 378.151 \quad / \sqrt{\ }$

$\boxed{b = 19.446 \text{ cm}}$

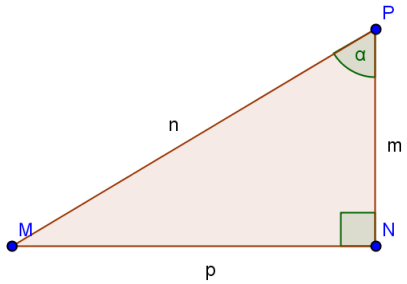
$a = 0.952b$

$\boxed{a = 18.513 \text{ cm}}$

$c^2 = a^2 + b^2$

$\boxed{c} = \sqrt{a^2 + b^2} = \sqrt{18.513^2 + 19.446^2} = \boxed{26.849 \text{ cm}}$

6. U pravokutnom trokutu MNP, u kojem je |MP| hipotenuza, vrijedi da je |NM| = 27 mm, a |NP| = 18 mm. Odredi vrijednost sinusa kuta kod vrha P.



skiciramo opisani pravokutni trokut i označimo traženi kut s α

$p = |MN| = |NM| = 27$

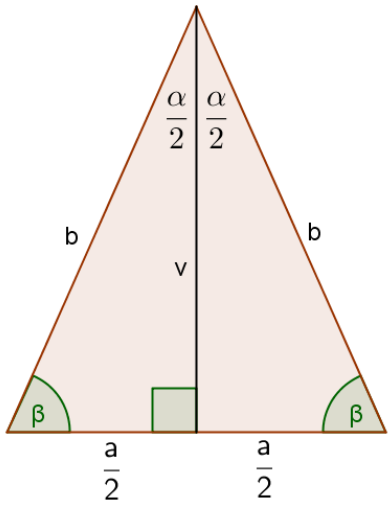
$m = |NP| = 18$

$n^2 = p^2 + m^2$

$n = \sqrt{p^2 + m^2} = \sqrt{27^2 + 18^2} = 32.45$

$\boxed{\sin\alpha} = \frac{p}{n} = \frac{27}{32.45} = \boxed{0.832}$

7. Izračunaj opseg i površinu jednakokračnog trokuta osnovice a = 58 cm i kuta nasuprot osnovice α = 76°16'.



$\sin\frac{\alpha}{2} = \frac{\frac{a}{2}}{b} = \frac{a}{2b}$

$2b = \frac{a}{\sin\frac{\alpha}{2}} = \frac{58}{\sin 38°8'} = 93.93$

$\boxed{b = 46.96\ \text{cm}}$

$\operatorname{tg}\frac{\alpha}{2} = \frac{\frac{a}{2}}{v} = \frac{a}{2v}$

$2v = \frac{a}{\operatorname{tg}\frac{\alpha}{2}} = \frac{58}{\operatorname{tg} 38°8'} = 73.88$

$\boxed{v = 36.94\ \text{cm}}$

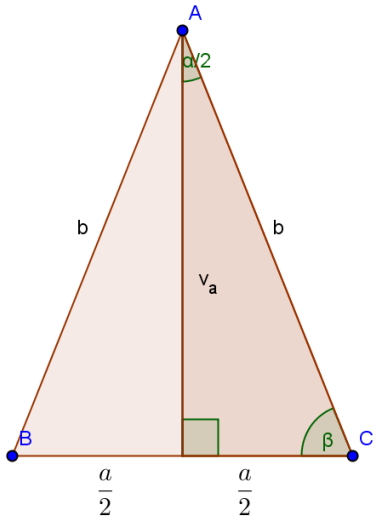
$O = a + 2b = 58 + 93.92$

$\boxed{O = 115.92\ \text{cm}}$

$P = \frac{a \cdot v}{2} = \frac{58 \cdot 36.94}{2}$

$\boxed{P = 1071.26\ cm^2}$

8. U jednakokračnom trokutu ABC duljina kraka je 24 cm, a osnovica je za četvrtinu te duljine kraća. Kolika je duljina visine iz vrha na osnovicu? Kolika je mjera kuta nasuprot osnovice?



$b = 24\ cm$

$a = 24 - \frac{24}{4} = 24 - 6 = 18\ cm$

$\boxed{v_a} = \sqrt{b^2 - \left(\frac{a}{2}\right)^2} = \sqrt{24^2 - 9^2} = \boxed{22.249}$

$\cos\frac{\alpha}{2} = \frac{v_a}{b}$

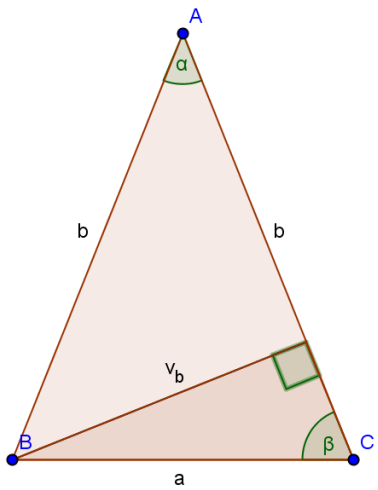
$\cos\frac{\alpha}{2} = \frac{22.249}{24}$

$\frac{\alpha}{2} = \cos^{-1}\left(\frac{22.249}{24}\right)$

$\frac{\alpha}{2} = 22°1'18''\quad /\cdot 2$

$\boxed{\alpha = 44°2'36''}$

9. Odredi mjere kutova jednakokračnog trokuta ako je duljina visine na krak v_b = 10, a osnovice a = 13.



$$\sin\beta = \frac{v_b}{a}$$

$$\sin\beta = \frac{10}{13}$$

$$\boxed{\beta} = \sin^{-1}\left(\frac{10}{13}\right) = \boxed{50°17'55''}$$

$$\alpha + 2\beta = 180°$$

$$\boxed{\alpha} = 180° - 2\beta = 180° - 100°35'50'' = \boxed{79°24'10''}$$

10. U jednakokračnom trokutu izrazi visinu na osnovicu v_a pomoću osnovice a i nasuprotnog kuta α.

skica je ista kao u zadatku 6.

$$\operatorname{tg}\frac{\alpha}{2} = \frac{\frac{a}{2}}{v_a}$$

$$v_a = \frac{\frac{a}{2}}{\operatorname{tg}\frac{\alpha}{2}}$$

$$\boxed{v_a = \frac{a}{2\operatorname{tg}\frac{\alpha}{2}}}$$

11. Omjer duljina kraka i visine na krak jednakokračnog trokuta iznosi $20:17$. Odredi mjeru kuta uz osnovicu.

skica je ista kao u zadatku 7.

$$b : v_b = 20 : 17$$

$$v_b : b = 17 : 20$$

$$\sin\alpha = \frac{v_b}{b}$$

$$\sin\alpha = \frac{17}{20}$$

$$\alpha = \sin^{-1}\left(\frac{17}{20}\right) = 58°12'42''$$

$$\alpha + 2\beta = 180°$$

$$2\beta = 180° - \alpha$$

$$2\beta = 180° - 58°12'42''$$

$$2\beta = 121°47'18'' \quad / : 2$$

$$\boxed{\beta = 60°53'39''}$$

12. Izračunaj duljine stranica jednakokračnog trokuta kojemu je opseg 270, a kut nasuprot osnovice 39°.

skica je ista kao u zadatku 6.

$$a + 2b = 270 \quad (*)$$

$$\sin\frac{\alpha}{2} = \frac{\frac{a}{2}}{b}$$

$$\sin\frac{\alpha}{2} = \frac{a}{2b}$$

$$a = 2b \cdot \sin\frac{\alpha}{2}$$

$$a = 0.668b$$

povratak u (*)

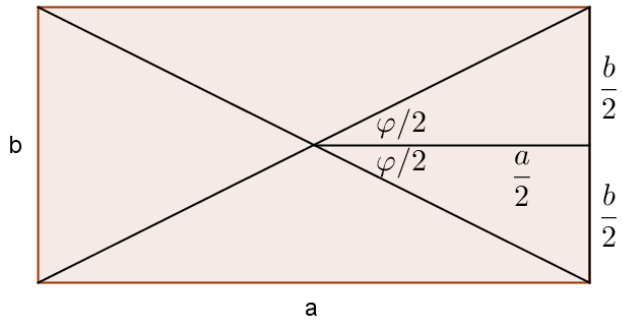
$$0.668b + 2b = 270$$

$$2.668b = 270 \quad / : 2.668$$

$$\boxed{b = 101.199}$$

$$\boxed{a} = 270 - 2b = \boxed{67.602}$$

13. Izračunaj površinu pravokutnika s duljom stranicom a = 14.3 cm i kutom među dijagonalama φ = 23°16'.



$$\operatorname{tg}\frac{\varphi}{2} = \frac{\frac{b}{2}}{\frac{a}{2}} = \frac{b}{a}$$

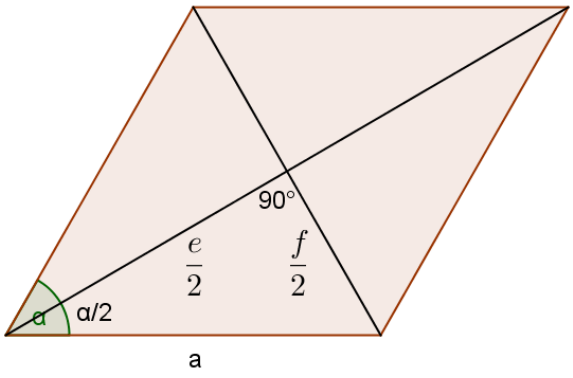
$$b = a \cdot \operatorname{tg}\frac{\varphi}{2} = 14.3 \cdot \operatorname{tg}11°38'$$

$$\boxed{b = 2.94 \text{ cm}}$$

$$P = a \cdot b = 14.3 \cdot 2.94$$

$$\boxed{P = 42.04 \text{ cm}^2}$$

14. Izračunaj šiljasti kut, stranicu i površinu romba čije su dijagonale duge 13 cm i 9.12 cm.



$$\operatorname{tg}\frac{\alpha}{2} = \frac{\frac{f}{2}}{\frac{e}{2}} = \frac{f}{e} = \frac{9.12}{13} = 0.70154$$

$$\frac{\alpha}{2} = 35°3'$$

$$\boxed{\alpha = 70°6'}$$

$$a^2 = \left(\frac{e}{2}\right)^2 + \left(\frac{f}{2}\right)^2$$

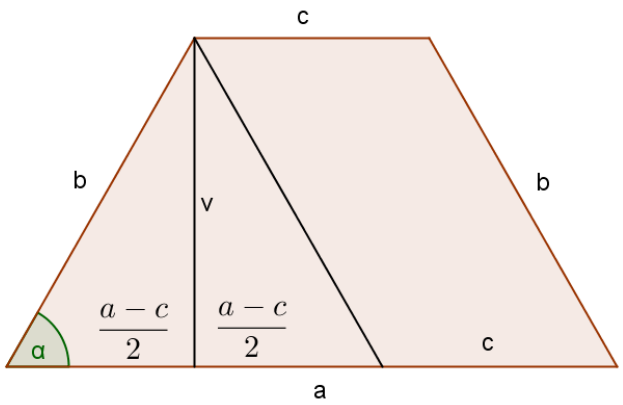
$$a = \sqrt{\left(\frac{e}{2}\right)^2 + \left(\frac{f}{2}\right)^2} = \sqrt{6.5^2 + 4.56^2} = \sqrt{42.25 + 20.7936} = \sqrt{63.0436}$$

$$\boxed{a = 7.94 \text{ cm}}$$

$$P = \frac{e \cdot f}{2} = \frac{13 \cdot 9.12}{2}$$

$$\boxed{P = 59.28 \text{ cm}^2}$$

15. Izračunaj površinu jednakokračnog trapeza s osnovicama duljine 19 i 13 cm, ako krak trapeza s osnovicom zatvara kut α = 67°28'.



$$\operatorname{tg}\alpha = \frac{v}{\frac{a-c}{2}}$$

$$v = \frac{a-c}{2} \cdot \operatorname{tg}\alpha = 3 \cdot \operatorname{tg} 67°28'$$

$$\boxed{v = 7.23 \text{ cm}}$$

$$P = \frac{a+c}{2} \cdot v = \frac{19+13}{2} \cdot 7.23$$

$$\boxed{P = 115.68 \text{ cm}^2}$$