

Ponavljanje za pismeni ispit – kvadratna jednadžba

1. Riješi jednadžbe:

a. $0,25x^2 + 100 = 0,$

$$\frac{25}{100}x^2 + 100 = 0$$

$$\frac{1}{4}x^2 + 100 = 0 \quad / \cdot 4$$

$$x^2 + 400 = 0$$

$$x^2 = -400 \quad / \sqrt{\quad}$$

$$\boxed{x_{1,2} = \pm 20i}$$

b. $(4x - 1)^2 - (3x + 1)^2 = 0,$

$$(4x)^2 - 2 \cdot 4x \cdot 1 + 1^2 - [(3x)^2 + 2 \cdot 3x \cdot 1 + 1^2] = 0$$

$$16x^2 - 8x + 1 - [9x^2 + 6x + 1] = 0$$

$$16x^2 - 8x + 1 - 9x^2 - 6x - 1 = 0$$

$$7x^2 - 14x = 0 \quad / : 7$$

$$x^2 - 2x = 0$$

$$x(x - 2) = 0$$

$$x = 0$$

$$\boxed{x_1 = 0}$$

$$x - 2 = 0$$

$$\boxed{x_2 = 2}$$

c. $\frac{x-2}{x} + \frac{x}{x+2} = 3.$

$$\frac{x-2}{x} + \frac{x}{x+2} = 3 \quad / \cdot x(x+2) \quad \begin{array}{l} x \neq 0 \\ x+2 \neq 0 \\ x \neq -2 \end{array}$$

$$(x-2)(x+2) + x^2 = 3x(x+2)$$

$$x^2 - 4 + x^2 = 3x^2 + 6x$$

$$2x^2 - 4 - 3x^2 - 6x = 0$$

$$-x^2 - 6x - 4 = 0 \quad / : (-1)$$

$$x^2 + 6x + 4 = 0$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-6 \pm \sqrt{6^2 - 4 \cdot 1 \cdot 4}}{2 \cdot 1} = \frac{-6 \pm \sqrt{36 - 16}}{2} = \frac{-6 \pm \sqrt{20}}{2} =$$

$$\frac{-6 \pm \sqrt{4 \cdot 5}}{2} = \frac{-6 \pm 2\sqrt{5}}{2} = \frac{2(-3 \pm \sqrt{5})}{2} = -3 \pm \sqrt{5}$$

$$\boxed{x_{1,2} = -3 \pm \sqrt{5}}$$

2. Za koji realni broj k jednadžba $x^2 + 2(k+2)x + k^2 + 2 = 0$ ima dvostruko rješenje?

$$a = 1, b = 2(k+2), c = k^2 + 2$$

$$D = b^2 - 4ac$$

$$D = [2(k+2)]^2 - 4 \cdot 1 \cdot (k^2 + 2) = 4(k+2)^2 - 4(k^2 + 2) = 4(k^2 + 4k + 4) - 4k^2 - 8 =$$

$$4k^2 + 16k + 16 - 4k^2 - 8 = 16k + 8$$

$$D = 0$$

$$16k + 8 = 0$$

$$16k = -8 \quad / : 16$$

$$k = -\frac{8}{16}$$

$$k = -\frac{1}{2}$$

3. Za koji realni broj m jednačba $mx^2 - 4mx + 4m - 1 = 0$ nema realnih rješenja?

$$a = m, b = -4m, c = 4m - 1$$

$$D = b^2 - 4ac$$

$$D = [-4m]^2 - 4 \cdot m \cdot (4m - 1) = 16m^2 - 16m^2 + 4m = 4m$$

$$D < 0$$

$$4m < 0 \quad / : 4$$

$$m < 0$$

$$m \in \langle -\infty, 0 \rangle$$

4. Ne rješavajući jednačbu $2x^2 - 3x - 6 = 0$, izračunaj:

a. $x_1^2 x_2 + x_1 x_2^2$,

$$x_1 + x_2 = -\frac{b}{a} = \frac{3}{2} \quad (*)$$

$$x_1 \cdot x_2 = \frac{c}{a} = \frac{-6}{2} = -3 \quad (**)$$

$$x_1^2 x_2 + x_1 x_2^2 = x_1 x_2 (x_1 + x_2) = -3 \cdot \frac{3}{2} = \boxed{-\frac{9}{2}}$$

b. $(x_1 + x_2)^{-2}$,

$$(x_1 + x_2)^{-2} = \frac{1}{(x_1 + x_2)^2} \stackrel{(*)}{=} \frac{1}{\left(\frac{3}{2}\right)^2} = \frac{1}{\frac{9}{4}} = \boxed{\frac{4}{9}}$$

c. $x_1^{-2} + x_2^{-2}$.

$$x_1^{-2} + x_2^{-2} = \frac{1}{x_1^2} + \frac{1}{x_2^2} = \frac{x_1^2 + x_2^2}{x_1^2 x_2^2} = \frac{x_1^2 + 2x_1 x_2 + x_2^2 - 2x_1 x_2}{(x_1 x_2)^2} =$$

$$\frac{(x_1 + x_2)^2 - 2x_1 x_2}{(x_1 x_2)^2} \stackrel{(*)}{=} \stackrel{(**)}{=} \frac{\left(\frac{3}{2}\right)^2 - 2(-3)}{(-3)^2} = \frac{\frac{9}{4} + 6}{9} = \frac{\frac{9+24}{4}}{9} = \frac{\frac{33}{4}}{9} = \frac{\frac{33}{4}}{\frac{9}{1}} = \frac{\frac{33}{4}}{\frac{9}{1}} = \frac{33}{4} \cdot \frac{1}{9} = \frac{11}{12} = \boxed{\frac{11}{12}}$$

5. Sastavi kvadratnu jednadžbu s cjelobrojnim koeficijentima kojoj je jedno rješenje

$$\frac{4}{2+i}$$

$$x_1 = \frac{4}{2+i} = \frac{4}{2+i} \cdot \frac{2-i}{2-i} = \frac{8-4i}{4+1} = \frac{8-4i}{5} = \frac{8}{5} - \frac{4}{5}i$$

$$x_2 = \frac{8}{5} + \frac{4}{5}i$$

$$x_1 + x_2 = \frac{8}{5} - \frac{4}{5}i + \frac{8}{5} + \frac{4}{5}i = \frac{16}{5}$$

$$x_1 \cdot x_2 = \left(\frac{8}{5} - \frac{4}{5}i\right)\left(\frac{8}{5} + \frac{4}{5}i\right) = \frac{64}{25} + \frac{16}{25} = \frac{80}{25} = \frac{16}{5}$$

$$x^2 - (x_1 + x_2)x + x_1x_2 = 0$$

$$x^2 - \frac{16}{5}x + \frac{16}{5} = 0 \quad / \cdot 5$$

$$\boxed{5x^2 - 16x + 16 = 0}$$

6. Skrati razlomak $\frac{10x^2 - x - 3}{50x^2 - 18}$.

$$10x^2 - x - 3 = 0$$

$$x_{1,2} = \frac{1 \pm \sqrt{(-1)^2 - 4 \cdot 10 \cdot (-3)}}{2 \cdot 10} = \frac{1 \pm \sqrt{1+120}}{20} = \frac{1 \pm \sqrt{121}}{20} = \frac{1 \pm 11}{20}$$

$$x_1 = \frac{1-11}{20} = \frac{-10}{20} = -\frac{1}{2}$$

$$x_2 = \frac{1+11}{20} = \frac{12}{20} = \frac{3}{5}$$

$$\frac{10x^2 - x - 3}{50x^2 - 18} = \frac{10\left(x - \left(-\frac{1}{2}\right)\right)\left(x - \frac{3}{5}\right)}{2(25x^2 - 9)} = \frac{2 \cdot 5\left(x + \frac{1}{2}\right)\left(x - \frac{3}{5}\right)}{2(5x-3)(5x+3)} = \frac{(2x+1)(5x-3)}{2(5x-3)(5x+3)} =$$

$$\boxed{\frac{(2x+1)}{2(5x+3)}}$$

7. Riješi jednadžbe:

a. $(x^2 + 2x)^2 - 14(x^2 + 2x) - 15 = 0$,

$$x^2 + 2x = t \quad (*)$$

$$t^2 - 14t - 15 = 0$$

$$t_{1,2} = \frac{14 \pm \sqrt{(-14)^2 - 4 \cdot 1 \cdot (-15)}}{2 \cdot 1} = \frac{14 \pm \sqrt{196 + 60}}{2} = \frac{14 \pm \sqrt{256}}{2} = \frac{14 \pm 16}{2}$$

$$t_1 = \frac{14 - 16}{2} = \frac{-2}{2} = -1$$

$$t_2 = \frac{14 + 16}{2} = \frac{30}{2} = 15$$

povratak u (*)

$$x^2 + 2x = -1 \qquad x^2 + 2x = 15$$

$$x^2 + 2x + 1 = 0 \qquad x^2 + 2x - 15 = 0$$

...

...

$$\boxed{x_{1,2} = -1}$$

$$\boxed{x_3 = -5, x_4 = 3}$$

b. $\frac{y^4}{6} - \frac{4y^2}{3} + 2 = 0,$

$$\frac{y^4}{6} - \frac{4y^2}{3} + 2 = 0 \quad / \cdot 6$$

$$y^4 - 8y^2 + 12 = 0$$

$$(y^2)^2 - 8y^2 + 12 = 0$$

$$y^2 = t \quad (*)$$

$$t^2 - 8t + 12 = 0$$

$$t_{1,2} = \frac{8 \pm \sqrt{(-8)^2 - 4 \cdot 1 \cdot 12}}{2 \cdot 1} = \frac{8 \pm \sqrt{64 - 48}}{2} = \frac{8 \pm \sqrt{16}}{2} = \frac{8 \pm 4}{2}$$

$$t_1 = \frac{8 - 4}{2} = \frac{4}{2} = 2$$

$$t_2 = \frac{8 + 4}{2} = \frac{12}{2} = 6$$

povratak u (*)

$$y^2 = 2 \quad / \sqrt{\quad}$$

$$\boxed{y_{1,2} = \pm\sqrt{2}}$$

$$y^2 = 6 \quad / \sqrt{\quad}$$

$$\boxed{y_{3,4} = \pm\sqrt{6}}$$

8. Riješi sustav $\begin{cases} x^2 + y^2 - 8x - 12y + 27 = 0 \\ x - 2y + 3 = 0 \end{cases}$

$$x - 2y + 3 = 0$$

$$x = 2y - 3 \quad (*)$$

uvrštavanje u prvu jednačbu

$$(2y - 3)^2 + y^2 - 8(2y - 3) - 12y + 27 = 0$$

$$4y^2 - 12y + 9 + y^2 - 16y + 24 - 12y + 27 = 0$$

$$5y^2 - 40y + 60 = 0 \quad / : 5$$

$$y^2 - 8y + 12 = 0$$

$$y_{1,2} = \frac{8 \pm \sqrt{(-8)^2 - 4 \cdot 1 \cdot 12}}{2 \cdot 1} = \frac{8 \pm \sqrt{64 - 48}}{2} = \frac{8 \pm \sqrt{16}}{2} = \frac{8 \pm 4}{2}$$

$$y_1 = \frac{8 - 4}{2} = \frac{4}{2} = 2$$

$$y_2 = \frac{8 + 4}{2} = \frac{12}{2} = 6$$

povratak u (*)

$$x_1 = 2y_1 - 3$$

$$x_2 = 2y_2 - 3$$

$$x_1 = 2 \cdot 2 - 3$$

$$x_2 = 2 \cdot 6 - 3$$

$$x_1 = 1$$

$$x_2 = 9$$

$$\boxed{(1, 2), (9, 6)}$$