Ponavljanje za pismeni ispit – potencije i korijeni

1. Izračunaj:

a. $4\sqrt{\frac{36}{49}}-\frac{1}{7}\sqrt{\frac{81}{25}}$,

$$4\sqrt{\frac{36}{49}}-\frac{1}{7}\sqrt{\frac{81}{25}}=4\cdot\frac{6}{7}-\frac{1}{7}\cdot\frac{9}{5}=\frac{24}{7}-\frac{9}{35}=\frac{120-9}{35}=\boxed{\frac{111}{35}}$$

b. $3\sqrt{0.25}-4\sqrt{0.0081}$,

$$3\sqrt{0.25}-4\sqrt{0.0081}=3\cdot0.5-4\cdot0.09=1.5-0.36=\boxed{1.14}$$

c. $\sqrt[3]{0.027}-2\sqrt[3]{0.008}$.

$$\sqrt[3]{0.027}-2\sqrt[3]{0.008}=0.3-2\cdot0.2=0.3-0.4=\boxed{-0.1}$$

d. $\frac{\sqrt[4]{80}}{\sqrt[4]{5}}+\sqrt[5]{4}\cdot\sqrt[4]{8}\cdot\sqrt[10]{8}\cdot\sqrt[20]{2^{11}}$,

$$\frac{\sqrt[4]{80}}{\sqrt[4]{5}}+\sqrt[5]{4}\cdot\sqrt[4]{8}\cdot\sqrt[10]{8}\cdot\sqrt[20]{2^{11}}=\sqrt[4]{\frac{80}{5}}+\sqrt[20]{4^4\cdot8^5\cdot8^2\cdot2^{11}}=$$

$$\sqrt[4]{16}+\sqrt[20]{\left(2^2\right)^4\cdot\left(2^3\right)^5\cdot\left(2^3\right)^2\cdot2^{11}}=2+\sqrt[20]{2^8\cdot2^{15}\cdot2^6\cdot2^{11}}=2+\sqrt[20]{2^{40}}=2+2^2=2+4$$

$$=\boxed{6}$$

e. $\dfrac{\left(\frac{1}{121}\right)^{-0.5}-16^{0.25}}{5\cdot8^{\frac{2}{3}}-8^{\frac{4}{3}}:32^{\frac{2}{5}}}$.

$$\frac{\left(\frac{1}{121}\right)^{-0.5}-16^{0.25}}{5\cdot8^{\frac{2}{3}}-8^{\frac{4}{3}}:32^{\frac{2}{5}}}=\frac{\left(11^{-2}\right)^{-\frac{1}{2}}-\left(2^4\right)^{\frac{1}{4}}}{5\cdot\left(2^3\right)^{\frac{2}{3}}-\left(2^3\right)^{\frac{4}{3}}:\left(2^5\right)^{\frac{2}{5}}}=\frac{11^{-2\cdot\left(-\frac{1}{2}\right)}-2^{4\cdot\frac{1}{4}}}{5\cdot2^{3\cdot\frac{2}{3}}-2^{3\cdot\frac{4}{3}}:2^{5\cdot\frac{2}{5}}}=\frac{11^1-2^1}{5\cdot2^2-2^4:2^2}=$$

$$\frac{11-2}{5\cdot4-16:4}=\frac{9}{20-4}=\boxed{\frac{9}{16}}$$

2. Djelomično korjenuj:

a. $\sqrt{\frac{12a^7b^3}{c^5}}$,

$$\sqrt{\frac{12a^7b^3}{c^5}}=\sqrt{\frac{4\cdot3\cdot a^6\cdot a\cdot b^2\cdot b}{c^4\cdot c}}=\boxed{\frac{2a^3b}{c^2}\sqrt{\frac{3ab}{c}}}$$

b. $\sqrt[3]{\frac{54x^6y^5}{z^4}}$.

$$\sqrt[3]{\frac{54x^6y^5}{z^4}}=\sqrt[3]{\frac{27\cdot2\cdot x^6\cdot y^3\cdot y^2}{z^3\cdot z}}=\boxed{\frac{3x^2y}{z}\sqrt[3]{\frac{2y^2}{z}}}$$

3. Unesi pod znak korijena:

a. $a^3\sqrt{a}$,

$$a^3\sqrt{a}=\sqrt{\left(a^3\right)^2\cdot a}=\sqrt{a^6\cdot a}=\boxed{\sqrt{a^7}}$$

b. $a^3\sqrt{a}$.

$$x\sqrt[3]{x^2}=\sqrt[3]{x^3\cdot x^2}=\boxed{\sqrt[3]{x^5}}$$

4. Ako su a i x pozitivni brojevi, svedi na jedan korijen:

a. $\sqrt[4]{a\sqrt[5]{a}}:\sqrt[10]{a}$,

$$\sqrt[4]{a\sqrt[5]{a}}:\sqrt[10]{a}=\sqrt[4]{\sqrt[5]{a^5\cdot a}}:\sqrt[10]{a}=\sqrt[20]{a^6}:\sqrt[10]{a}=\sqrt[20]{a^6}:\sqrt[20]{a^2}=\sqrt[20]{a^6:a^2}=\sqrt[20]{a^4}=\boxed{\sqrt[5]{a}}$$

b. $\left(x^{\frac{2}{3}}\right)^{-\frac{9}{4}}\cdot\left(x^{\frac{1}{4}}\right)^{3}$.

$$\left(x^{\frac{2}{3}}\right)^{-\frac{9}{4}}\cdot\left(x^{\frac{1}{4}}\right)^{3}=x^{\frac{2}{3}\cdot\left(-\frac{9}{4}\right)}\cdot x^{\frac{1}{4}\cdot 3}=x^{\frac{1}{1}\cdot\left(-\frac{3}{2}\right)}\cdot x^{\frac{3}{4}}=x^{-\frac{3}{2}}\cdot x^{\frac{3}{4}}=x^{-\frac{3}{2}+\frac{3}{4}}=x^{\frac{-6+3}{4}}=x^{\frac{-3}{4}}=\boxed{\sqrt[4]{x^{-3}}}$$

5. Izračunaj:

a. $4\sqrt{160}-7\sqrt{810}+\sqrt{4000}$,

$$4\sqrt{160}-7\sqrt{810}+\sqrt{4000}=4\sqrt{16\cdot 10}-7\sqrt{81\cdot 10}+\sqrt{400\cdot 10}=$$
$$4\cdot 4\sqrt{10}-7\cdot 9\sqrt{10}+20\sqrt{10}=16\sqrt{10}-63\sqrt{10}+20\sqrt{10}=\boxed{-27\sqrt{10}}$$

b. $12\sqrt[3]{54}-3\sqrt[3]{16}+\sqrt[3]{250}$.

$$12\sqrt[3]{54}-3\sqrt[3]{16}+\sqrt[3]{250}=12\sqrt[3]{27\cdot 2}-3\sqrt[3]{8\cdot 2}+\sqrt[3]{125\cdot 2}=$$
$$12\cdot 3\sqrt[3]{2}-3\cdot 2\sqrt[3]{2}+5\sqrt[3]{2}=36\sqrt[3]{2}-6\sqrt[3]{2}+5\sqrt[3]{2}=\boxed{35\sqrt[3]{2}}$$

6. Izračunaj:

a. $\sqrt{72}-\left(\sqrt{6}-2\sqrt{3}\right)^2$,

$$\sqrt{72}-\left(\sqrt{6}-2\sqrt{3}\right)^2=\sqrt{72}-\left(\left(\sqrt{6}\right)^2-2\cdot\sqrt{6}\cdot 2\sqrt{3}+\left(2\sqrt{3}\right)^2\right)=$$
$$\sqrt{72}-\left(6-4\sqrt{18}+4\cdot 3\right)=\sqrt{72}-\left(6-4\sqrt{18}+12\right)=\sqrt{72}-\left(18-4\sqrt{18}\right)=$$
$$\sqrt{72}-18+4\sqrt{18}=\sqrt{36\cdot 2}-18+4\sqrt{9\cdot 2}=6\sqrt{2}-18+4\cdot 3\sqrt{2}=$$
$$6\sqrt{2}-18+12\sqrt{2}=\boxed{18\sqrt{2}-18}$$

b. $\left(\sqrt{5}-1\right)^2\left(6+2\sqrt{5}\right)$,

$$\left(\sqrt{5}-1\right)^2\left(6+2\sqrt{5}\right)=\left(\left(\sqrt{5}\right)^2-2\cdot\sqrt{5}\cdot 1+1^2\right)\left(6+2\sqrt{5}\right)=\left(5-2\sqrt{5}+1\right)\left(6+2\sqrt{5}\right)=$$

$$\left(6-2\sqrt{5}\right)\left(6+2\sqrt{5}\right)=6^2-\left(2\sqrt{5}\right)^2=36-4\cdot 5=36-20=\boxed{16}$$

c. $\sqrt[3]{4+\sqrt{7}}\cdot\sqrt[3]{4-\sqrt{7}}\cdot\sqrt[3]{3}$,

$$\sqrt[3]{4+\sqrt{7}}\cdot\sqrt[3]{4-\sqrt{7}}\cdot\sqrt[3]{3}=\sqrt[3]{\left(4+\sqrt{7}\right)\left(4-\sqrt{7}\right)}\cdot\sqrt[3]{3}=\sqrt[3]{4^2-\left(\sqrt{7}\right)^2}\cdot\sqrt[3]{3}=$$

$$\sqrt[3]{16-7}\cdot\sqrt[3]{3}=\sqrt[3]{9}\cdot\sqrt[3]{3}=\sqrt[3]{9\cdot 3}=\sqrt[3]{27}=\boxed{3}$$

d. $\left(\sqrt[3]{\frac{x^2}{y^2}}-\sqrt[3]{\frac{x}{y}}+1\right)\left(\sqrt[3]{\frac{x}{y}}+1\right)$,

$$\left(\sqrt[3]{\frac{x^2}{y^2}}-\sqrt[3]{\frac{x}{y}}+1\right)\left(\sqrt[3]{\frac{x}{y}}+1\right)=\left(\sqrt[3]{\frac{x}{y}}+1\right)\left(\sqrt[3]{\frac{x^2}{y^2}}-\sqrt[3]{\frac{x}{y}}+1\right)=\left(\sqrt[3]{\frac{x}{y}}\right)^3+1^3=\boxed{\frac{x}{y}+1}$$

7. Racionaliziraj:

a. $\frac{2}{\sqrt[3]{2}}$,

$$\frac{2}{\sqrt[3]{2}}=\frac{2}{\sqrt[3]{2}}\cdot\frac{\sqrt[3]{2^2}}{\sqrt[3]{2^2}}=\frac{2\sqrt[3]{2^2}}{\sqrt[3]{2^3}}=\frac{2\sqrt[3]{2^2}}{2}=\sqrt[3]{2^2}=\boxed{\sqrt[3]{4}}$$

b. $\frac{1}{\sqrt[5]{2\sqrt[3]{2}}}$,

$$\frac{1}{\sqrt[5]{2\sqrt[3]{2}}}=\frac{1}{\sqrt[5]{\sqrt[3]{2^3\cdot 2}}}=\frac{1}{\sqrt[15]{2^4}}=\frac{1}{\sqrt[15]{2^4}}\cdot\frac{\sqrt[15]{2^{11}}}{\sqrt[15]{2^{11}}}=\frac{\sqrt[15]{2^{11}}}{\sqrt[15]{2^{15}}}=\boxed{\frac{\sqrt[15]{2^{11}}}{2}}$$

c. $\frac{2\sqrt{3}-\sqrt{2}}{2\sqrt{3}+\sqrt{2}}$,

$$\frac{2\sqrt{3}-\sqrt{2}}{2\sqrt{3}+\sqrt{2}}=\frac{2\sqrt{3}-\sqrt{2}}{2\sqrt{3}+\sqrt{2}}\cdot\frac{2\sqrt{3}-\sqrt{2}}{2\sqrt{3}-\sqrt{2}}=\frac{\left(2\sqrt{3}-\sqrt{2}\right)^2}{\left(2\sqrt{3}+\sqrt{2}\right)\left(2\sqrt{3}-\sqrt{2}\right)}=$$

$$\frac{\left(2\sqrt{3}\right)^2-2\cdot 2\sqrt{3}\cdot\sqrt{2}+\left(\sqrt{2}\right)^2}{\left(2\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2}=\frac{4\cdot 3-4\sqrt{6}+2}{4\cdot 3-2}=\frac{12-4\sqrt{6}+2}{12-2}=\frac{14-4\sqrt{6}}{10}=$$

$$\frac{2\left(7-2\sqrt{6}\right)}{10}=\boxed{\frac{7-2\sqrt{6}}{5}}$$

d. $\frac{7}{\sqrt[4]{8}-1}$,

$$\frac{7}{\sqrt[4]{8}-1}=\frac{7}{\sqrt[4]{8}-1}\cdot\frac{\sqrt[4]{8}+1}{\sqrt[4]{8}+1}=\frac{7\left(\sqrt[4]{8}+1\right)}{\left(\sqrt[4]{8}\right)^2-1^2}=\frac{7\left(\sqrt[4]{8}+1\right)}{\sqrt[4]{8^2}-1}=\frac{7\left(\sqrt[4]{8}+1\right)}{\sqrt{8}-1}=\frac{7\left(\sqrt[4]{8}+1\right)}{\sqrt{8}-1}\cdot\frac{\sqrt{8}+1}{\sqrt{8}+1}$$

$$=\frac{7\left(\sqrt[4]{8}+1\right)\left(\sqrt{8}+1\right)}{\left(\sqrt{8}\right)^2-1^2}=\frac{7\left(\sqrt[4]{8}+1\right)\left(\sqrt{8}+1\right)}{8-1}=\frac{7\left(\sqrt[4]{8}+1\right)\left(\sqrt{8}+1\right)}{7}=\boxed{\left(\sqrt[4]{8}+1\right)\left(\sqrt{8}+1\right)}$$

8. Pojednostavni:

a. $\left(\frac{\sqrt[4]{a}}{\sqrt[4]{a}-\sqrt[4]{b}}-\frac{\sqrt[4]{b}}{\sqrt[4]{a}+\sqrt[4]{b}}\right):\frac{1}{a-b}$,

$$\left(\frac{\sqrt[4]{a}}{\sqrt[4]{a}-\sqrt[4]{b}}-\frac{\sqrt[4]{b}}{\sqrt[4]{a}+\sqrt[4]{b}}\right):\frac{1}{a-b}=\frac{\sqrt[4]{a}\left(\sqrt[4]{a}+\sqrt[4]{b}\right)-\sqrt[4]{b}\left(\sqrt[4]{a}-\sqrt[4]{b}\right)}{\left(\sqrt[4]{a}-\sqrt[4]{b}\right)\left(\sqrt[4]{a}+\sqrt[4]{b}\right)}:\frac{1}{a-b}=$$

$$\frac{\left(\sqrt[4]{a}\right)^2+\sqrt[4]{a}\sqrt[4]{b}-\sqrt[4]{a}\sqrt[4]{b}+\left(\sqrt[4]{b}\right)^2}{\left(\sqrt[4]{a}\right)^2-\left(\sqrt[4]{b}\right)^2}:\frac{1}{a-b}=\frac{\sqrt[4]{a^2}+\sqrt[4]{b^2}}{\sqrt[4]{a^2}-\sqrt[4]{b^2}}:\frac{1}{a-b}=\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}}:\frac{1}{a-b}=$$

$$\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}}\cdot\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}+\sqrt{b}}:\frac{1}{a-b}=\frac{\left(\sqrt{a}+\sqrt{b}\right)^2}{\left(\sqrt{a}\right)^2-\left(\sqrt{b}\right)^2}:\frac{1}{a-b}=\frac{\left(\sqrt{a}+\sqrt{b}\right)^2}{a-b}\cdot\frac{a-b}{1}=$$

$$\boxed{\left(\sqrt{a}+\sqrt{b}\right)^2}$$

b. $\left[\frac{a^{\frac{1}{2}}}{2}\left(\frac{1}{a^{\frac{1}{2}}-1}+\frac{1}{a^{\frac{1}{2}}+1}\right)-\frac{a}{a+1}\right]\cdot a^{-\frac{1}{2}}$,

$$\left[\frac{a^{\frac{1}{2}}}{2}\left(\frac{1}{a^{\frac{1}{2}}-1}+\frac{1}{a^{\frac{1}{2}}+1}\right)-\frac{a}{a+1}\right]\cdot a^{-\frac{1}{2}}=\left[\frac{\sqrt{a}}{2}\left(\frac{1}{\sqrt{a}-1}+\frac{1}{\sqrt{a}+1}\right)-\frac{a}{a+1}\right]\cdot\frac{1}{a^{\frac{1}{2}}}=$$

$$\left[\frac{\sqrt{a}}{2}\cdot\frac{\sqrt{a}+1+\sqrt{a}-1}{\left(\sqrt{a}-1\right)\left(\sqrt{a}+1\right)}-\frac{a}{a+1}\right]\cdot\frac{1}{\sqrt{a}}=\left[\frac{\sqrt{a}}{2}\cdot\frac{2\sqrt{a}}{\left(\sqrt{a}\right)^2-1^2}-\frac{a}{a+1}\right]\cdot\frac{1}{\sqrt{a}}=$$

$$\left[\frac{\sqrt{a}}{1}\cdot\frac{\sqrt{a}}{a-1}-\frac{a}{a+1}\right]\cdot\frac{1}{\sqrt{a}}=\left[\frac{\left(\sqrt{a}\right)^2}{a-1}-\frac{a}{a+1}\right]\cdot\frac{1}{\sqrt{a}}=\left[\frac{a}{a-1}-\frac{a}{a+1}\right]\cdot\frac{1}{\sqrt{a}}=$$

$$\frac{a(a+1)-a(a-1)}{(a-1)(a+1)}\cdot\frac{1}{\sqrt{a}}=\frac{a^2+a-a^2+a}{(a-1)(a+1)}\cdot\frac{1}{\sqrt{a}}=\frac{2a}{(a-1)(a+1)}\cdot\frac{1}{\sqrt{a}}\cdot\frac{\sqrt{a}}{\sqrt{a}}=$$

$$\frac{2a}{(a-1)(a+1)}\cdot\frac{\sqrt{a}}{a}=\frac{2}{(a-1)(a+1)}\cdot\frac{\sqrt{a}}{1}=\boxed{\frac{2\sqrt{a}}{(a-1)(a+1)}}$$

9. Zlatar je 27 zlatnih kockica s bridom duljine 2 mm pretopio u jednu kocku. Kolika je duljina brida nove kocke?

Obujam nove kocke mora biti jednak zbroju obujama 27 malih kocaka.

$V_m=2^3\ mm^3=8\ mm^3$

$V_v=27\cdot 8\ mm^3=216\ mm^3$

$V_v=a^3$

$a^3 = 216$

$a = \sqrt[3]{216}$

$a = 6\,\text{mm}$

Duljina brida nove kocke iznosi $\boxed{6\text{ mm}}$.