Ponavljanje za pismeni ispit – eksponencijalna i logaritamska funkcija

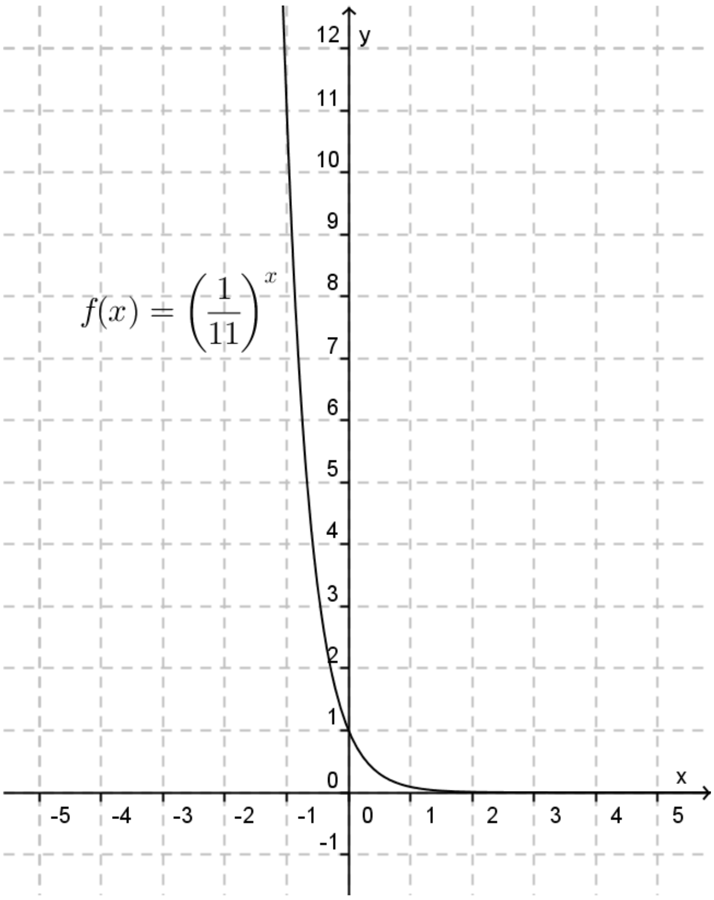
1. Zadana je funkcija $f(x)=\left(\frac{1}{11}\right)^{5x}\cdot 11^{4x+2}\cdot\frac{1}{121}$.

   a. Pojednostavni f(x).

$$f(x)=\left(11^{-1}\right)^{5x}\cdot 11^{4x+2}\cdot 11^{-2}$$
$$f(x)=11^{-5x}\cdot 11^{4x+2}\cdot 11^{-2}$$
$$f(x)=11^{-5x+4x+2-2}$$
$$f(x)=11^{-x}$$
$$\boxed{f(x)=\left(\frac{1}{11}\right)^{x}}$$

   b. Skiciraj graf funkcije f.

x	$f(x)=\left(\frac{1}{11}\right)^{x}$
-1	11
0	1
1	$\frac{1}{11}$



   c. Sa slike očitaj domenu i kodomenu (sliku) te rast odnosno pad funkcije. U kojim točkama graf presjeca koordinatne osi?

$\boxed{D=R}$

$R = \langle 0, \infty \rangle$

Funkcija f strogo pada na cijeloj domeni.

Graf ne siječe x-os, a presjek s y-osi je točka (0,1).

d. Pripada li točka $(2, \frac{1}{11})$ grafu funkcije f?

$$f(x) = \left(\frac{1}{11}\right)^x$$

$$(2, \frac{1}{11})$$

$$f(2) = \left(\frac{1}{11}\right)^2 = \frac{1}{121} \neq \frac{1}{11}$$

Koordinate točke ne zadovoljavaju jednakost kojom je zadana funkcija.

Točka ne pripada grafu.

e. Odredi i na grafu funkcije f označi točku T kojoj je ordinata jednaka 5. Rezultat zaokruži na 3 decimale.

$$f(x) = \left(\frac{1}{11}\right)^x$$

$$y = 5$$

$$f(x) = 5$$

$$\left(\frac{1}{11}\right)^x = 5 \ /\log$$

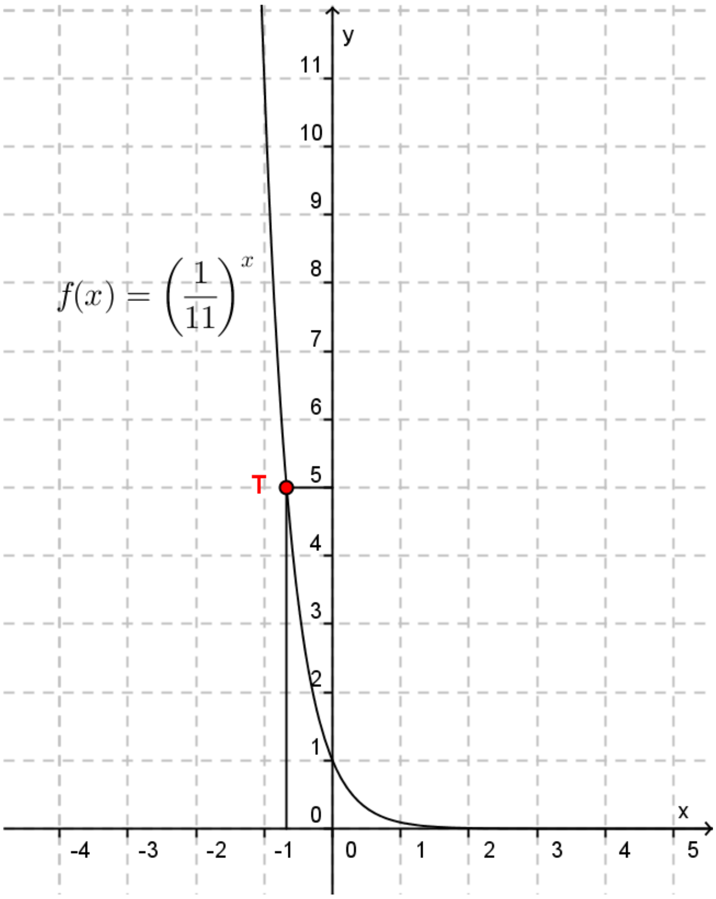
$$\log\left(\frac{1}{11}\right)^x = \log 5$$

$$x \log\left(\frac{1}{11}\right) = \log 5 \ / : \log\left(\frac{1}{11}\right)$$

$$x = \frac{\log 5}{\log\left(\frac{1}{11}\right)}$$

$$x = -0.671$$

$$T(-0.671, 5)$$



f. Za koje je vrijednosti x ispunjena nejednakost $f(x) \le \sqrt[7]{121^5}$?

$$f(x) = \left(\frac{1}{11}\right)^x$$

$$f(x) \le \sqrt[7]{121^5}$$

$$\left(\frac{1}{11}\right)^x \le \sqrt[7]{121^5}$$

$$\left(\frac{1}{11}\right)^x \le 121^{\frac{5}{7}}$$

$$\left(\frac{1}{11}\right)^x \le \left(\left(\frac{1}{11}\right)^{-2}\right)^{\frac{5}{7}}$$

$$\left(\frac{1}{11}\right)^x \le \left(\frac{1}{11}\right)^{-\frac{10}{7}}$$

$$x \ge -\frac{10}{7}$$

$$\boxed{x \in \left[-\frac{10}{7}, \infty\right\rangle}$$

g. Izračunaj $f(\log_{11} 8 - 1)$.

$$f(x) = \left(\frac{1}{11}\right)^x$$

$$f(\log_{11}8-1)=\left(\frac{1}{11}\right)^{\log_{11}8-1}=\left(\frac{1}{11}\right)^{\log_{11}8-\log_{11}11}=\left(\frac{1}{11}\right)^{\log_{11}\frac{8}{11}}=\left(11^{-1}\right)^{\log_{11}\frac{8}{11}}=$$

$$=\left(11^{\log_{11}\frac{8}{11}}\right)^{-1}=\left(\frac{8}{11}\right)^{-1}=\frac{11}{8}$$

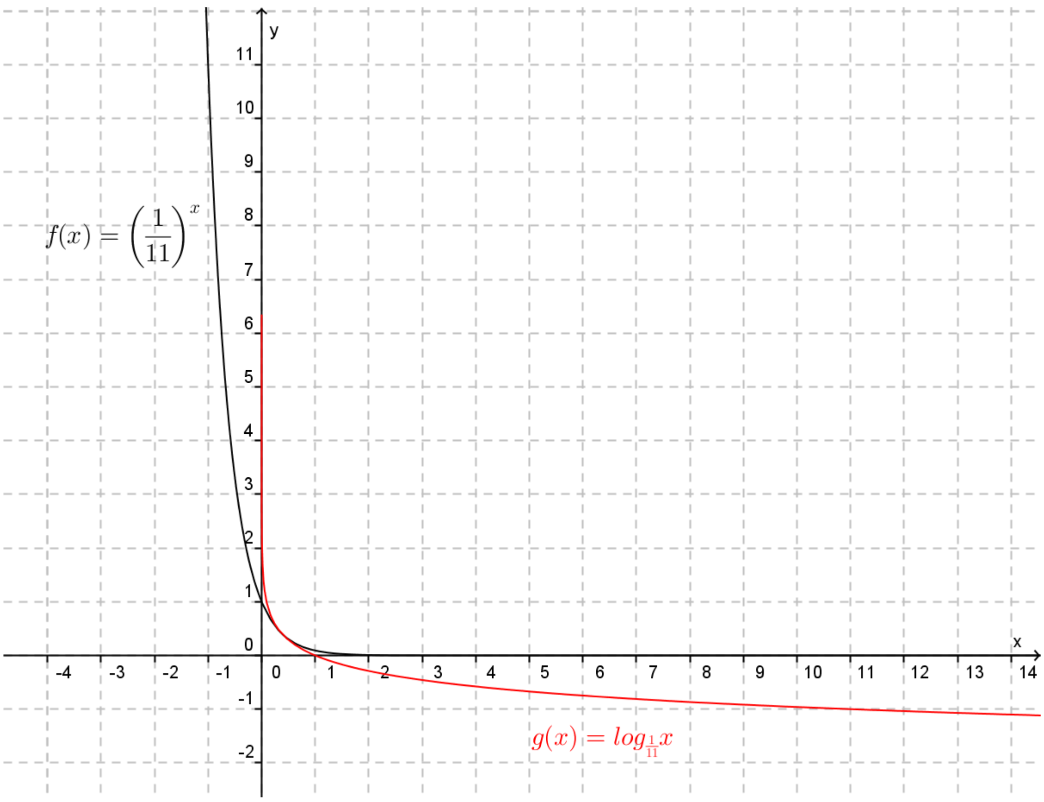
$$\boxed{f(\log_{11}8-1)=\frac{11}{8}}$$

h. Odredi inverznu funkciju funkcije f. Označi je s f^{-1}, odnosno s g.

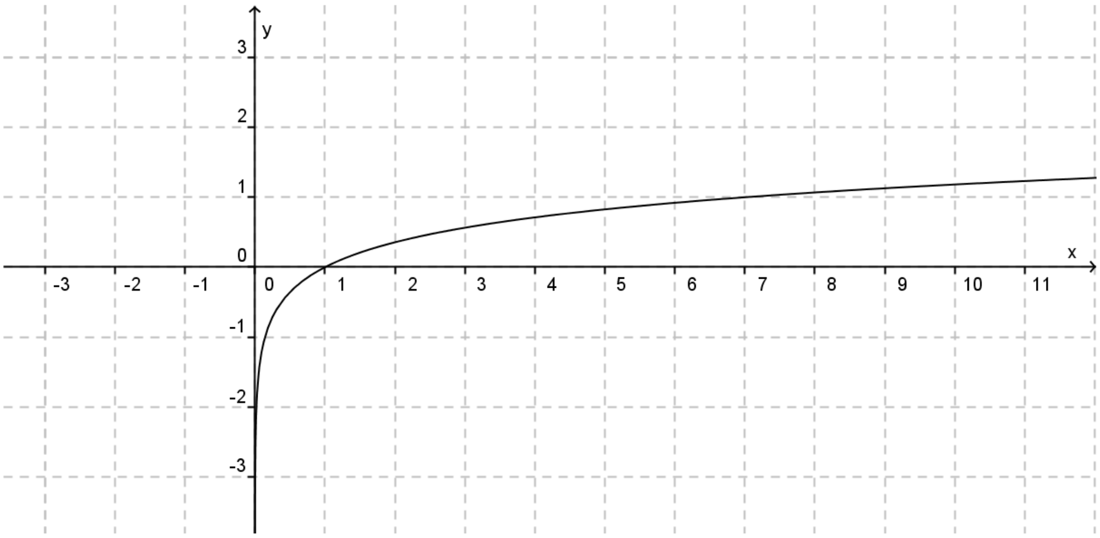
$$\boxed{f^{-1}(x)=g(x)=\log_{\frac{1}{11}}x}$$

i. Skiciraj graf funkcije g u istom koordinatnom sustavu, kao i f.

x	$g(x)=\log_{\frac{1}{11}}x$
11	-1
1	0
$\frac{1}{11}$	1



2. Na slici je prikazan graf logaritamske funkcije $f(x) = \log_a x$.



a. Odredi bazu a funkcije f.

sa slike se vidi da graf funkcije, osim kroz točku (1,0), prolazi i kroz točku (7,1)

$f(x) = \log_a x$

$1 = \log_a 7$

$a^1 = 7$

$\boxed{a = 7}$

$\boxed{f(x) = \log_7 x}$

b. Odredi i na grafu funkcije f označi točku T kojoj je apscisa jednaka 5. Rezultat zaokruži na 3 decimale.

$f(x) = \log_7 x$

$x = 5$

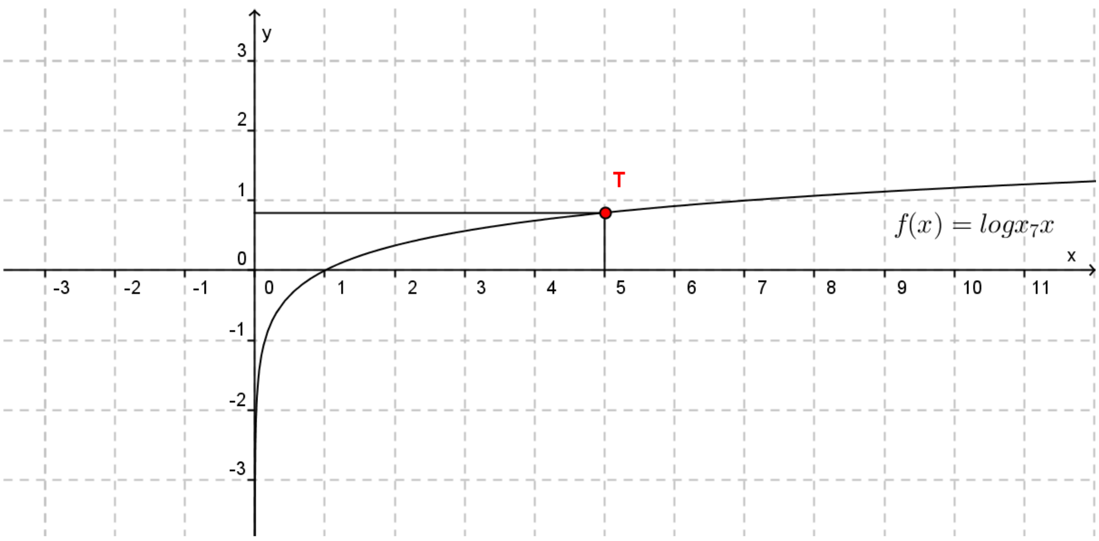
$f(5) = \log_7 5$

$$f(5) = \frac{\log 5}{\log 7}$$

$f(5) = 0.827$

$y = 0.827$

$\boxed{T(5, 0.827)}$



c. Izračunaj $f\left(\left(\frac{1}{49}\right)^{-\sqrt{17}}\right)$.

$f(x) = \log_7 x$

$f(x) = \log_7\left(\frac{1}{49}\right)^{-\sqrt{17}} = \log_7\left(7^{-2}\right)^{-\sqrt{17}} \log_7 7^{2\sqrt{17}} = 2\sqrt{17}$

$$\boxed{f\left(\left(\frac{1}{49}\right)^{-\sqrt{17}}\right) = 2\sqrt{17}}$$

d. Ako je $f(x) = \log_7 5 - \frac{1}{2}\log_7 2 + 1$, koliki je x?

$f(x) = \log_7 x$

$\log_7 x = \log_7 5 - \frac{1}{2}\log_7 2 + 1$

$\log_7 x = \log_7 5 - \log_7 2^{\frac{1}{2}} + \log_7 7$

$\log_7 x = \log_7 5 + \log_7 7 - \log_7 \sqrt{2}$

$\log_7 x = \log_7 35 - \log_7 \sqrt{2}$

$\log_7 x = \log_7 \frac{35}{\sqrt{2}}$

$$\boxed{x = \frac{35}{\sqrt{2}} = \frac{35\sqrt{2}}{2}}$$

3. Riješi jednadžbe i nejednadžbe:

a. $\left(5^{-2} \cdot \sqrt[5]{125}\right)^{2x} = 0.04^{x-1}$,

$\left(5^{-2} \cdot \sqrt[5]{5^3}\right)^{2x} = \left(\frac{4}{100}\right)^{x-1}$

$\left(5^{-2} \cdot 5^{\frac{3}{5}}\right)^{2x} = \left(\frac{1}{25}\right)^{x-1}$

$\left(5^{-2+\frac{3}{5}}\right)^{2x} = \left(5^{-2}\right)^{x-1}$

$\left(5^{\frac{-10+3}{5}}\right)^{2x} = 5^{-2x+2}$

$\left(5^{-\frac{7}{5}}\right)^{2x} = 5^{-2x+2}$

$5^{-\frac{14}{5}x} = 5^{-2x+2}$

$-\frac{14}{5}x = -2x + 2 \ /\cdot 5$

$-14x = -10x + 10$
$-14 + 10x = 10$
$-4x = 10 \;/ : (-4)$

$x = -\dfrac{10}{4}$

$\boxed{x = -\dfrac{5}{2}}$

b. $5^x - 5^{-2+x} = 120$,

$5^x - 5^{-2} \cdot 5^x = 120$

$5^x - \dfrac{1}{25} \cdot 5^x = 120 \;/ \cdot 25$

$25 \cdot 5^x - 5^x = 3000$

$5^x (25 - 1) = 3000$

$5^x \cdot 24 = 3000 \;/ : 24$

$5^x = 125$

$5^x = 5^3$

$\boxed{x = 3}$

c. $4^{x+1} - 129 \cdot 2^x + 32 = 0$,

$4^1 \cdot 4^x - 129 \cdot 2^x + 32 = 0$

$4 \cdot (2^x)^2 - 129 \cdot 2^x + 32 = 0$

$2^x = t$

$4t^2 - 129t + 32 = 0$

$t_{1,2} = \dfrac{129 \pm \sqrt{16641 - 512}}{8} = \dfrac{129 \pm \sqrt{16129}}{8} = \dfrac{129 \pm 127}{8}$

$t_1 = \dfrac{129 + 127}{8} = \dfrac{256}{8} = 32$

$t_2 = \dfrac{129 - 127}{8} = \dfrac{2}{8} = \dfrac{1}{4}$

$2^x = 32$ $\qquad\qquad$ $2^x = \dfrac{1}{4}$

$2^x = 2^5$ $\qquad\qquad$ $2^x = 2^{-2}$

$\boxed{x_1 = 5}$ $\qquad\qquad$ $\boxed{x_2 = -2}$

d. $6 \cdot 81^{2x-3} < 2$,

$6 \cdot 81^{2x-3} < 2 \;/ : 6$

$81^{2x-3} < \dfrac{2}{6}$

$81^{2x-3} < \dfrac{1}{3}$

$(3^4)^{2x-3} < 3^{-1}$

$3^{8x-12} < 3^{-1}$
$8x - 12 < -1$
$8x < -1 + 12$

$$8x < 11 \; / : 8$$

$$x < \frac{11}{8}$$

$$\boxed{x \in \left\langle -\infty, \frac{11}{8} \right\rangle}$$

e. $5^x \geq 3$,

$$5^x \geq 3 \; / \log$$

$$\log 5^x \geq \log 3$$

$$x \log 5 \geq \log 3 \; / : \log 5 > 0$$

$$x \geq \frac{\log 3}{\log 5}$$

$$x \geq 1.465$$

$$\boxed{x \in [1.465, \infty\rangle}$$

f. $\log(5x + 15) - 1 = 2\log x$,

uvjeti:

$5x + 15 > 0$ $\qquad$ $x > 0$

$$5x > -15 \; / : 5$$

$$x > -3$$

$$\log(5x + 15) - \log 10 = \log x^2$$

$$\log \frac{5x + 15}{10} = \log x^2$$

$$\frac{5x + 15}{10} = x^2 \; / \cdot 10$$

$$5x + 15 = 10x^2$$

$$-10x^2 + 5x + 15 = 0 \; / : (-5)$$

$$2x^2 - x - 3 = 0$$

$$x_{1,2} = \frac{1 \pm \sqrt{1 + 24}}{4} = \frac{1 \pm \sqrt{25}}{4} = \frac{1 \pm 5}{4}$$

$$\boxed{x_1 = \frac{6}{4} = \frac{3}{2}}$$

$x_2 = \frac{-4}{4} = -1$ (ne prihvaćamo, jer ne zadovoljava drugi uvjet)

g. $\log_5 \log_3 \log_{\frac{1}{2}} x = \log_5 10 - 1$,

uvjet: $\quad x > 0$

$$\log_5 \log_3 \log_{\frac{1}{2}} x = \log_5 10 - \log_5 5$$

$$\log_5 \log_3 \log_{\frac{1}{2}} x = \log_5 \frac{10}{5}$$

$$\log_5 \log_3 \log_{\frac{1}{2}} x = \log_5 2$$

$$\log_3 \log_{\frac{1}{2}} x = 2$$

$$\log_{\frac{1}{2}} x = 3^2$$

$$\log_{\frac{1}{2}} x = 9$$

$$x = \left(\frac{1}{2}\right)^9$$

$$\boxed{x = \frac{1}{512}}$$