Ponavljanje za pismeni ispit – grafovi trigonometrijskih funkcija

1. Skiciraj graf funkcije $f(x) = 2\sin(\frac{x}{3})$.

$a = 2, b = \frac{1}{3}, c = 0$

Amplituda: $A = |a| = 2$

Temeljni period: $T = \frac{2\pi}{|b|} = \frac{2\pi}{\frac{1}{3}} = 6\pi$

Nultočke: $\frac{x}{3} = 0 \quad / \cdot 3$

$x_1 = 0$

$x_3 = x_1 + T = 0 + 6\pi = 6\pi$

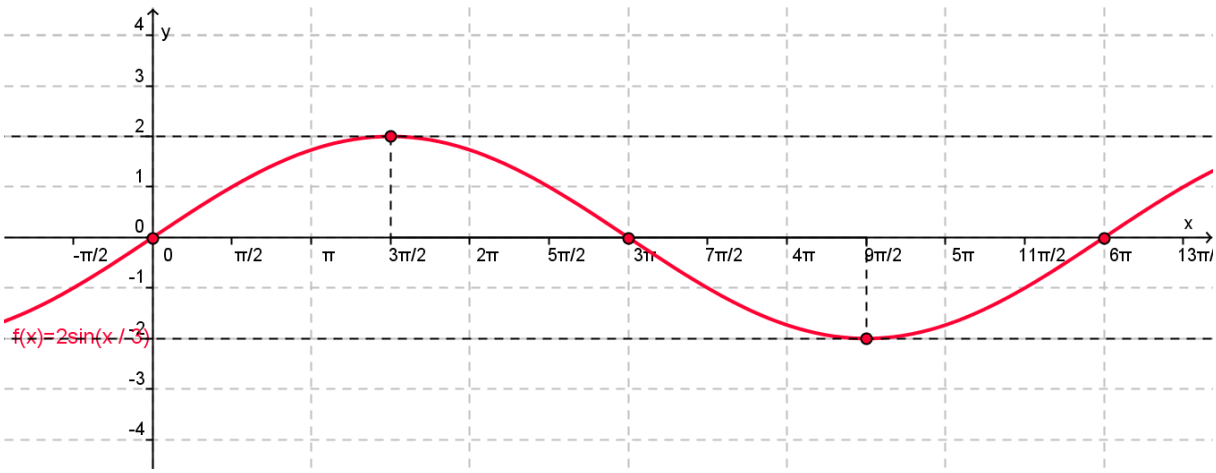
$x_2 = \frac{x_1 + x_3}{2} = \frac{0 + 6\pi}{2} = 3\pi$

Maksimum: $M = |a| = |2| = 2$

$a > 0 \Rightarrow x_{max} = \frac{x_1 + x_2}{2} = \frac{0 + 3\pi}{2} = \frac{3\pi}{2}$

Minimum: $m = -|a| = -|2| = -2$

$a > 0 \Rightarrow x_{min} = \frac{x_2 + x_3}{2} = \frac{3\pi + 6\pi}{2} = \frac{9\pi}{2}$



2. Skiciraj graf funkcije $f(x) = -\frac{1}{2}\cos(x + \frac{3\pi}{4})$.

$a = -\frac{1}{2}, b = 1, c = \frac{3\pi}{4}$

Amplituda: $A = |a| = \frac{1}{2}$

Temeljni period: $T = \frac{2\pi}{|b|} = \frac{2\pi}{1} = 2\pi$

Nultočke: $x + \frac{3\pi}{4} = \frac{\pi}{2}$

$x = \frac{\pi}{2} - \frac{3\pi}{4}$

$x = \frac{2\pi - 3\pi}{4}$

$x_1 = -\frac{\pi}{4}$

$x_3 = x_1 + T = -\frac{\pi}{4} + 2\pi = -\frac{\pi}{4} + \frac{2\pi}{1} = \frac{-\pi + 8\pi}{4} = \frac{7\pi}{4}$

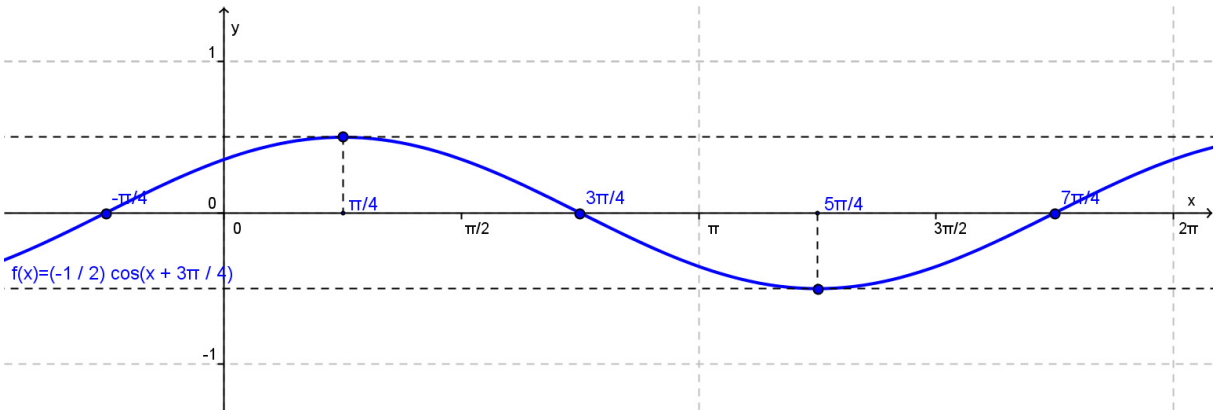
$x_2 = \frac{x_1 + x_3}{2} = \frac{-\frac{\pi}{4} + \frac{7\pi}{4}}{2} = \frac{\frac{6\pi}{4}}{\frac{2}{1}} = \frac{6\pi}{8} = \frac{3\pi}{4}$

Maksimum: $M = |a| = \left|-\frac{1}{2}\right| = \frac{1}{2}$

$a < 0 \Rightarrow x_{max} = \frac{x_1 + x_2}{2} = \frac{-\frac{\pi}{4} + \frac{3\pi}{4}}{2} = \frac{\frac{2\pi}{4}}{\frac{2}{1}} = \frac{2\pi}{8} = \frac{\pi}{4}$

Minimum: $m = -|a| = -\left|-\frac{1}{2}\right| = -\frac{1}{2}$

$a < 0 \Rightarrow x_{min} = \frac{x_2 + x_3}{2} = \frac{\frac{3\pi}{4} + \frac{7\pi}{4}}{2} = \frac{\frac{10\pi}{4}}{\frac{2}{1}} = \frac{10\pi}{8} = \frac{5\pi}{4}$



3. Skiciraj graf funkcije $f(x) = -\text{tg}(3x)$.

$a = -1, b = 3, c = 0$

Temeljni period: $T = \frac{\pi}{|b|} = \frac{\pi}{3}$

Nultočke: $3x = 0 \quad / : 3$

$x_1 = 0$

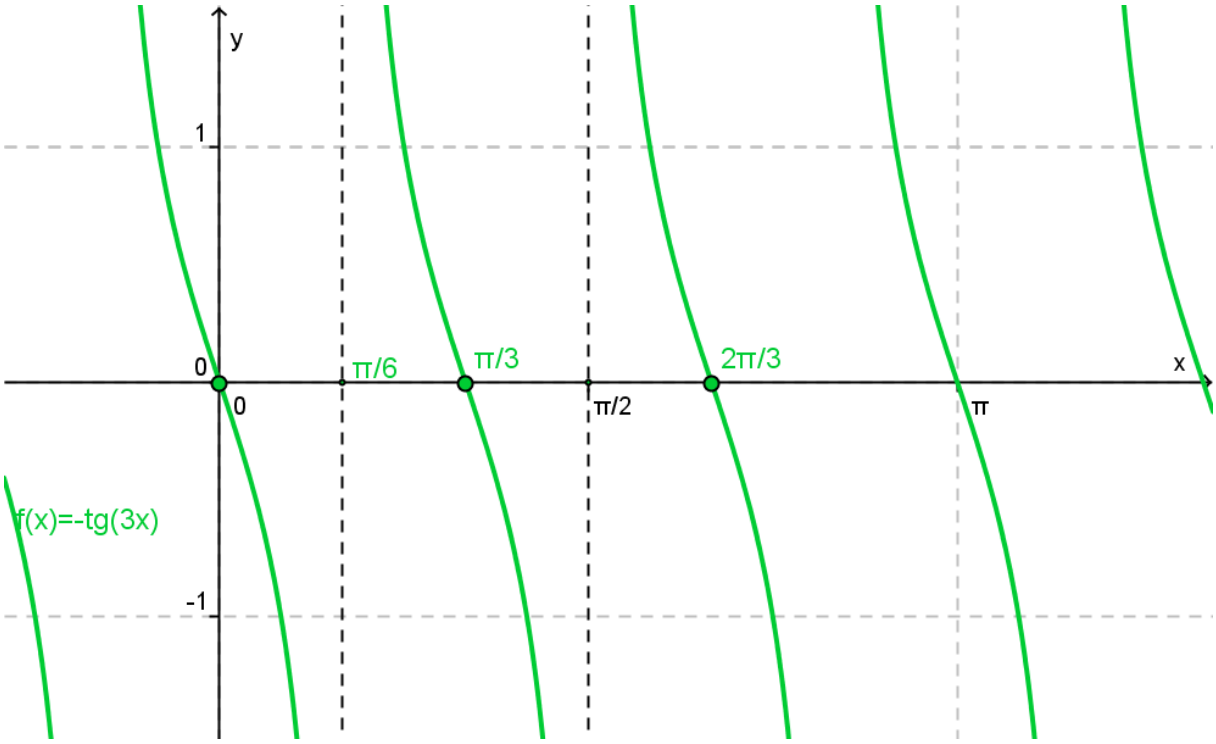
$x_2 = x_1 + T = 0 + \frac{\pi}{3} = \frac{\pi}{3}$

$x_3 = x_2 + T = \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}$

Asimptote: $x_{A1} = \frac{x_1 + x_2}{2} = \frac{0 + \frac{\pi}{3}}{2} = \frac{\frac{\pi}{3}}{\frac{2}{1}} = \frac{\pi}{6}$

$x_{A2} = \frac{x_2 + x_3}{2} = \frac{\frac{\pi}{3} + \frac{2\pi}{3}}{2} = \frac{\frac{3\pi}{3}}{\frac{2}{1}} = \frac{3\pi}{6} = \frac{\pi}{2}$

Pad ili rast: $a < 0 \Rightarrow$ funkcija pada na intervalima



4. Skiciraj graf funkcije $f(x)=\frac{1}{5}\operatorname{ctg}(\frac{1}{4}x+\frac{\pi}{3})$.

$a=\frac{1}{5}, b=\frac{1}{4}, c=\frac{\pi}{3}$

Temeljni period: $T=\frac{\pi}{|b|}=\frac{\pi}{\frac{1}{4}}=4\pi$

Nultočke:
$$\frac{1}{4}x+\frac{\pi}{3}=\frac{\pi}{2}$$
$$\frac{1}{4}x=\frac{\pi}{2}-\frac{\pi}{3}$$
$$\frac{1}{4}x=\frac{3\pi-2\pi}{6}$$
$$\frac{1}{4}x=\frac{\pi}{6} \quad /\cdot 4$$
$$x_1=\frac{4\pi}{6}=\frac{2\pi}{3}$$
$$x_2=x_1+T=\frac{2\pi}{3}+4\pi=\frac{2\pi}{3}+\frac{4\pi}{1}=\frac{2\pi+12\pi}{3}=\frac{14\pi}{3}$$
$$x_3=x_2+T=\frac{14\pi}{3}+4\pi=\frac{14\pi}{3}+\frac{4\pi}{1}=\frac{14\pi+12\pi}{3}=\frac{26\pi}{3}$$

Asimptote:
$$x_{A1}=\frac{x_1+x_2}{2}=\frac{\frac{2\pi}{3}+\frac{14\pi}{3}}{2}=\frac{\frac{16\pi}{3}}{\frac{2}{1}}=\frac{16\pi}{6}=\frac{8\pi}{3}$$
$$x_{A2}=\frac{x_2+x_3}{2}=\frac{\frac{14\pi}{3}+\frac{26\pi}{3}}{2}=\frac{\frac{40\pi}{3}}{\frac{2}{1}}=\frac{40\pi}{6}=\frac{20\pi}{3}$$

Pad / rast: $a>0 \Rightarrow$ funkcija pada na intervalima