

KRUŽNICA

- 1) Kako glasi jednačica kružnice kojoj je točka $S(0, -1)$ središte, a prolazi točkom $A(1, 2)$?

$$S(0, -1), A(1, 2)$$

$$p \ q, \quad x \ y$$

$$(x-p)^2 + (y-q)^2 = r^2$$

$$(1-0)^2 + (2+1)^2 = r^2$$

$$1^2 + 3^2 = r^2$$

$$1 + 9 = r^2$$

$$r^2 = 10$$

$$(x-0)^2 + (y+1)^2 = 10$$

$$\boxed{x^2 + (y+1)^2 = 10}$$

- 2) Napiši jednačicu kružnice koja prolazi točkama $A(3, 4)$, $B(0, 5)$, $C(4, -3)$.

$$(x-p)^2 + (y-q)^2 = r^2$$

$$(3-p)^2 + (4-q)^2 = r^2$$

$$(0-p)^2 + (5-q)^2 = r^2$$

$$(4-p)^2 + (-3-q)^2 = r^2$$

$$\begin{aligned} (-3-q)^2 &= (3+q)^2 \\ &= 9 + 6q + q^2 \end{aligned}$$

$$9 - 6p + p^2 + 16 - 8q + q^2 = r^2$$

$$p^2 + 25 - 10q + q^2 = r^2$$

$$16 - 8p + p^2 + 9 + 6q + q^2 = r^2$$

$$p^2 - 6p + q^2 - 8q + 25 = r^2$$

$$p^2 + q^2 - 10q + 25 = r^2 \quad | \cdot (-1)$$

$$p^2 - 8p + q^2 + 6q + 25 = r^2$$

$$p^2 - 6p + q^2 - 8q + 25 = r^2 \quad | +$$

$$-p^2 - q^2 + 10q - 25 = -r^2 \quad | +$$

$$p^2 - 8p + q^2 + 6q + 25 = r^2 \quad | +$$

$$-6p + 2q = 0 \quad | : 2$$

$$-8p + 16q = 0 \quad | : 8$$

$$-3p + 2q = 0 \quad | \cdot (-2)$$

$$-p + 2q = 0$$

$$6p - 2q = 0 \quad | +$$

$$-p + 2q = 0 \quad | +$$

$$5p = 0 \quad | : 5$$

$$\boxed{p = 0}$$

$$0 + 2q = 0$$

$$2q = 0 \quad | : 2$$

$$\boxed{q = 0}$$

$$p^2 + 25 - 10q + q^2 = r^2$$

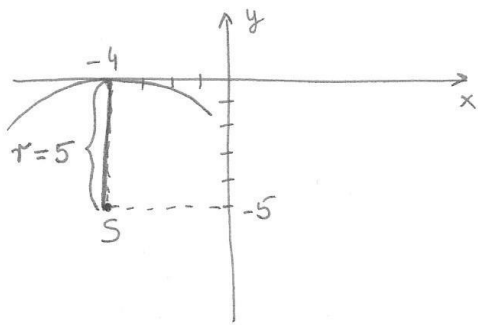
$$25 = r^2$$

$$\boxed{r^2 = 25}$$

$$(x-p)^2 + (y-q)^2 = r^2$$

$$\boxed{x^2 + y^2 = 25}$$

3) Kružnice sa središtem u $S(-4, -5)$ diti x -os. Napiši jednačinu!



$$(x-p)^2 + (y-q)^2 = r^2$$

$$(x+4)^2 + (y+5)^2 = 5^2$$

$$(x+4)^2 + (y+5)^2 = 25$$

4) Odredi jednačinu kružnice koja prolazi tačkama $A(4, 4)$ i $B(-3, 3)$, a središte joj je na x -osi.

$$S(p, 0)$$

$$(x-p)^2 + (y-q)^2 = r^2$$

$$(4-p)^2 + (4-0)^2 = r^2$$

$$(-3-p)^2 + (3-0)^2 = r^2$$

$$16 - 8p + p^2 + 16 = r^2$$

$$9 + 6p + p^2 + 9 = r^2$$

$$p^2 - 8p + 32 = r^2$$

$$p^2 + 6p + 18 = r^2 \cdot (-1)$$

$$p^2 - 8p + 32 = r^2$$

$$-p^2 - 6p - 18 = -r^2 \quad | +$$

$$-14p + 14 = 0$$

$$-14p = -14 \quad | :(-14)$$

$$p = 1$$

$$1 - 8 + 32 = r^2$$

$$25 = r^2$$

$$r^2 = 25$$

$$(x-p)^2 + (y-q)^2 = r^2$$

$$(x-1)^2 + y^2 = 25$$

5) Odredi jednačinu kružnice poluprijeka $r=9$, koncentrične kružnici

$$x^2 + y^2 + 2x + 2y - 14 = 0.$$

$$x^2 + y^2 + 2x + 2y - 14 = 0$$

$$x^2 + 2x + y^2 + 2y - 14 = 0$$

$$\underbrace{x^2 + 2x + 1}_{(x+1)^2} - 1 + \underbrace{y^2 + 2y + 1}_{(y+1)^2} - 1 - 14 = 0$$

$$(x+1)^2 + (y+1)^2 - 16 = 0$$

$$(x+1)^2 + (y+1)^2 = 16$$

$$S(-1, -1)$$

$$S(-1, -1), r=9$$

$$(x-p)^2 + (y-q)^2 = r^2$$

$$(x+1)^2 + (y+1)^2 = 81$$

- 6) Odredi međusobni položaj pravca $y = -x + 11$ i kružnice $(x-1)^2 + y^2 = 16$.

$$\begin{array}{l} (x-1)^2 + y^2 = 16 \\ y = -x + 11 \end{array}$$

$$\begin{aligned} (x-1)^2 + (-x+11)^2 &= 16 \\ x^2 - 2x + 1 + x^2 - 22x + 121 &= 16 \\ x^2 - 2x + 1 + x^2 - 22x + 121 - 16 &= 0 \\ 2x^2 - 24x + 106 &= 0 \quad /: 2 \\ x^2 - 12x + 53 &= 0 \end{aligned}$$

$$(-x+11)^2 = (x-11)^2 = x^2 - 22x + 121$$

$$D = b^2 - 4ac$$

$$D = (-12)^2 - 4 \cdot 1 \cdot 53$$

$$D = 144 - 212$$

$$D = -68 < 0$$

kwadratna jednačina nema realnih rješenja \Rightarrow

pravac i kružnica nemaju zajedničkih tačaka

- 7) Odredi dužinu tetive kružnice $x^2 + y^2 = 16$ određene pravcem $y = -x$.

$$\begin{array}{l} x^2 + y^2 = 16 \\ y = -x \end{array}$$

$$\begin{aligned} x^2 + (-x)^2 &= 16 \\ x^2 + x^2 &= 16 \\ 2x^2 &= 16 \quad /: 2 \\ x^2 &= 8 \quad \sqrt{} \\ x &= \pm\sqrt{8} = \pm 2\sqrt{2} \end{aligned}$$

$$x_1 = 2\sqrt{2}$$

$$x_2 = -2\sqrt{2}$$

$$y_1 = -2\sqrt{2}$$

$$y_2 = 2\sqrt{2}$$

$$S_1(2\sqrt{2}, -2\sqrt{2}), S_2(-2\sqrt{2}, 2\sqrt{2})$$

$$\begin{aligned} t = |S_1 S_2| &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-2\sqrt{2} - 2\sqrt{2})^2 + (2\sqrt{2} + 2\sqrt{2})^2} = \\ &= \sqrt{(-4\sqrt{2})^2 + (4\sqrt{2})^2} = \sqrt{16 \cdot 2 + 16 \cdot 2} = \sqrt{32 + 32} = \sqrt{64} = 8 \end{aligned}$$

$$t = 8$$

8) Napiši jednačinu tangente i normale na kružnicu $(x+1)^2 + (y-3)^2 = 169$ u njenoj tački $D(x > -1, -9)$.

$$(x+1)^2 + (y-3)^2 = 169$$

$$(x+1)^2 + (-9-3)^2 = 169$$

$$(x+1)^2 + (-12)^2 = 169$$

$$(x+1)^2 = 169 - 144$$

$$(x+1)^2 = 25 \quad \sqrt{\quad}$$

$$x+1 = \pm 5$$

$$x_1 = 5-1 \quad x_2 = -5-1$$

$$x_1 = 4 \quad x_2 = -6$$

$$D(4, -9)$$

$$(x_0-p)(x-p) + (y_0-q)(y-q) = r^2$$

$$(4+1)(x+1) + (-9-3)(y-3) = 169$$

$$5(x+1) - 12(y-3) = 169$$

$$5x+5 - 12y+36 = 169$$

$$-12y = -5x + 169 - 5 - 36$$

$$-12y = -5x + 128 \quad | :(-12)$$

$$y = \frac{5}{12}x - \frac{128}{12}$$

$$t \dots \boxed{y = \frac{5}{12}x - \frac{32}{3}}$$

$$k_t = \frac{5}{12}, \quad k_n = -\frac{12}{5}, \quad D(4, -9)$$

$$y - y_1 = k(x - x_1)$$

$$y + 9 = -\frac{12}{5}(x - 4)$$

$$y + 9 = -\frac{12}{5}x + \frac{48}{5}$$

$$y = -\frac{12}{5}x + \frac{48}{5} - 9$$

$$y = -\frac{12}{5}x + \frac{48-45}{5}$$

$$n \dots \boxed{y = -\frac{12}{5}x + \frac{3}{5}}$$

9) Odredi jednačine tangenti kružnice $(x-2)^2 + (y-2)^2 = 25$ paralelnih pravcu $3x - 4y - 1 = 0$.

$$3x - 4y - 1 = 0$$

$$-4y = -3x + 1 \quad | :(-4)$$

$$y = \frac{3}{4}x - \frac{1}{4}$$

$$k = \frac{3}{4}$$

$$(x-2)^2 + (y-2)^2 = 25$$

$$p = 2$$

$$q = 2$$

$$r^2 = 25$$

$$r^2(k^2+1) = (kp-q+l)^2$$

$$25\left(\frac{9}{16}+1\right) = \left(\frac{3}{4} \cdot \frac{3}{4} - 2 + l\right)^2$$

$$25 \cdot \frac{25}{16} = \left(\frac{9}{16} - 2 + l\right)^2$$

$$\frac{25 \cdot 25}{16} = \left(\frac{3-4}{2} + l\right)^2$$

$$\frac{625}{16} = \left(l - \frac{1}{2}\right)^2$$

$$\left(l - \frac{1}{2}\right)^2 = \frac{625}{16} \quad \sqrt{\quad}$$

$$l - \frac{1}{2} = \pm \frac{25}{4}$$

$$l - \frac{1}{2} = \frac{25}{4}$$

$$l = \frac{25}{4} + \frac{1}{2}$$

$$l = \frac{25+2}{4}$$

$$l_1 = \frac{27}{4}$$

$$t_1 \dots \boxed{y = \frac{3}{4}x + \frac{27}{4}}$$

$$l - \frac{1}{2} = -\frac{25}{4}$$

$$l = -\frac{25}{4} + \frac{1}{2}$$

$$l = \frac{-25+2}{4}$$

$$l_2 = -\frac{23}{4}$$

$$t_2 \dots \boxed{y = \frac{3}{4}x - \frac{23}{4}}$$

10) Napiši rovnice tangenti kružnice $x^2 + y^2 - 14y + 32 = 0$,
 povrchem iz tocke $T(5,4)$.

$$x^2 + y^2 - 14y + 32 = 0$$

$$x^2 + y^2 - 14y + 49 - 49 + 32 = 0$$

$$x^2 + (y-7)^2 - 17 = 0$$

$$x^2 + (y-7)^2 = 17$$

$$p=0, q=7, r^2=17$$

$$y = kx + l$$

$$T(5,4)$$

$$4 = k \cdot 5 + l$$

$$4 = 5k + l$$

$$5k + l = 4$$

$$l = 4 - 5k$$

$$r^2(k^2+1) = (kp - q + l)^2$$

$$17(k^2+1) = (k \cdot 0 - 7 + 4 - 5k)^2$$

$$17k^2 + 17 = (-5k - 3)^2$$

$$17k^2 + 17 = (5k + 3)^2$$

$$17k^2 + 17 = 25k^2 + 30k + 9$$

$$17k^2 + 17 - 25k^2 - 30k - 9 = 0$$

$$-8k^2 - 30k + 8 = 0 \quad /: (-2)$$

$$4k^2 + 15k - 4 = 0$$

$$k_{1,2} = \frac{-15 \pm \sqrt{225 + 64}}{8} = \frac{-15 \pm \sqrt{289}}{8} = \frac{-15 \pm 17}{8}$$

$$k_1 = \frac{-15 + 17}{8} = \frac{2}{8} = \frac{1}{4} \quad , \quad k_2 = \frac{-15 - 17}{8} = \frac{-32}{8} = -4$$

$$l_1 = 4 - 5 \cdot \frac{1}{4}$$

$$= 4 - \frac{5}{4}$$

$$= \frac{16 - 5}{4}$$

$$= \frac{11}{4}$$

$$l_2 = 4 - 5 \cdot (-4)$$

$$= 4 + 20$$

$$= 24$$

$$t_1 \dots \boxed{y = \frac{1}{4}x + \frac{11}{4}}$$

$$t_2 \dots \boxed{y = -4x + 24}$$