Ponavljanje za pismeni ispit – brojevi

1. Zapiši u kraćem obliku:
   a. $101 \cdot 100 \cdot 99 \cdot 98!$,

   $101 \cdot 100 \cdot 99 \cdot 98! = \boxed{101!}$

   b. $(n-1)! \cdot n \cdot (n+1) \cdot (n+2) \cdot (n+3)$.

   $(n-1)! \cdot n \cdot (n+1) \cdot (n+2) \cdot (n+3) = \boxed{(n+3)!}$

2. Skrati razlomke:
   a. $\dfrac{30!}{28!}$,

   $\dfrac{30!}{28!} = \dfrac{28! \cdot 29 \cdot 30}{28!} = 29 \cdot 30 = \boxed{870}$

   b. $\dfrac{(n-2)!}{(n+1)!}$.

   $\dfrac{(n-2)!}{(n+1)!} = \dfrac{(n-2)!}{(n-2)! \cdot (n-1) \cdot n \cdot (n+1)} = \boxed{\dfrac{1}{(n-1) \cdot n \cdot (n+1)}}$

3. Riješi jednadžbu $\dfrac{(k+1)!}{(k-1)!} = 30$, gdje je k prirodni broj.

   $\dfrac{(k-1)! \cdot k \cdot (k+1)}{(k-1)!} = 30$

   $k \cdot (k+1) = 30$

   $k^2 + k - 30 = 0$

   ...

   $\boxed{k_1 = 5}$

   $k_2 = -6$ ne prihvaćamo

4. Izračunaj:
   a. $\binom{200}{3}$,

   $\binom{200}{3} = \dfrac{200 \cdot 199 \cdot 198}{1 \cdot 2 \cdot 3} = \boxed{1313400}$

   b. $\binom{200}{198}$.

   $\binom{200}{198} = \binom{200}{2} = \dfrac{200 \cdot 199}{1 \cdot 2} = \boxed{19900}$

5. Računom provjeri da vrijedi:

   a. $\dbinom{12}{8} = \dbinom{12}{4}$,

$$\frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8} = \frac{12 \cdot 11 \cdot 10 \cdot 9}{1 \cdot 2 \cdot 3 \cdot 4}$$

$$495 = 495$$

   b. $\dbinom{10}{3} + \dbinom{10}{4} = \dbinom{11}{4}$.

$$\frac{10 \cdot 9 \cdot 8}{1 \cdot 2 \cdot 3} + \frac{10 \cdot 9 \cdot 8 \cdot 7}{1 \cdot 2 \cdot 3 \cdot 4} = \frac{11 \cdot 10 \cdot 9 \cdot 8}{1 \cdot 2 \cdot 3 \cdot 4}$$

$$120 + 210 = 330$$

$$330 = 330$$

6. Riješi jednadžbu $2\dbinom{n}{4} = \dbinom{n+1}{4}$, gdje je n prirodni broj.

$$2\frac{n \cdot (n-1) \cdot (n-2) \cdot (n-3)}{1 \cdot 2 \cdot 3 \cdot 4} = \frac{(n+1) \cdot n \cdot (n-1) \cdot (n-2)}{1 \cdot 2 \cdot 3 \cdot 4} \;/\cdot 24$$

$$2n \cdot (n-1) \cdot (n-2) \cdot (n-3) = (n+1) \cdot n \cdot (n-1) \cdot (n-2) \;/: n(n-1)(n-2)$$

$$2(n-3) = n+1$$

$$2n - 6 = n + 1$$

$$2n - n = 6 + 1$$

$$\boxed{n = 7}$$

7. Pomoću binomnog poučka raspiši $\left(x^2 - \dfrac{2}{x}\right)^5$.

$$\left(x^2 - \frac{2}{x}\right)^5 = \binom{5}{0}(x^2)^5\left(-\frac{2}{x}\right)^0 + \binom{5}{1}(x^2)^4\left(-\frac{2}{x}\right)^1 + \binom{5}{2}(x^2)^3\left(-\frac{2}{x}\right)^2 +$$

$$\binom{5}{3}(x^2)^2\left(-\frac{2}{x}\right)^3 + \binom{5}{4}(x^2)^1\left(-\frac{2}{x}\right)^4 + \binom{5}{5}(x^2)^0\left(-\frac{2}{x}\right)^5 =$$

$$1 \cdot x^{10} \cdot 1 + 5 \cdot x^8 \cdot \left(-\frac{2}{x}\right) + 10 \cdot x^6 \cdot \frac{4}{x^2} + 10 \cdot x^4 \cdot \left(-\frac{8}{x^3}\right) + 5 \cdot x^2 \cdot \frac{16}{x^4} + 1 \cdot 1 \cdot \left(-\frac{32}{x^5}\right) =$$

$$\boxed{x^{10} - 10x^7 + 40x^4 - 80x + \frac{80}{x^2} - \frac{32}{x^5}}$$

8. Odredi:

   a. sedmi član u razvoju $(2x+1)^9$,

$$T_7 = \binom{9}{6}(2x)^3 \cdot 1^6 = \frac{9 \cdot 8 \cdot 7}{1 \cdot 2 \cdot 3} \cdot 8x^3 = \boxed{672x^3}$$

b. koeficijent uz x^5 u razvoju $\left(\sqrt{x} - \sqrt{3}\right)^{12}$,

$$T_{k+1} = \binom{12}{k}\left(\sqrt{x}\right)^{12-k}\left(\sqrt{3}\right)^{k} = \binom{12}{k}\sqrt{x^{12-k}} \cdot \sqrt{3^k} = \binom{12}{k}x^{\frac{12-k}{2}} \cdot 3^{\frac{k}{2}}$$

$$x^{\frac{12-k}{2}} = x^5$$

$$\frac{12-k}{2} = 5 \;/\cdot 2$$

$$12 - k = 10$$

$$-k = -2$$

$$k = 2$$

$$T_3 = \binom{12}{2}x^5 \cdot 3^1 = 66x^5 \cdot 3 = 198x^5$$

traženi koeficijent iznosi $\boxed{198}$

c. slobodni član (član bez x) u razvoju $\left(\sqrt{x^3} + \dfrac{1}{\sqrt{x}}\right)^{8}$.

$$\left(\sqrt{x^3} + \frac{1}{\sqrt{x}}\right)^{8} = \left(x^{\frac{3}{2}} + x^{-\frac{1}{2}}\right)^{8}$$

$$T_{k+1} = \binom{8}{k}\left(x^{\frac{3}{2}}\right)^{8-k}\left(x^{-\frac{1}{2}}\right)^{k} = \binom{8}{k}x^{\frac{3}{2}(8-k)} \cdot x^{-\frac{k}{2}} = \binom{8}{k}x^{12-\frac{3}{2}k} \cdot x^{-\frac{k}{2}} =$$

$$\binom{8}{k}x^{12-\frac{3k}{2}-\frac{k}{2}} = \binom{8}{k}x^{12-\frac{4k}{2}} = \binom{8}{k}x^{12-2k}$$

$$x^{12-2k} = x^0$$

$$12 - 2k = 0$$

$$-2k = -12 \;/ : (-2)$$

$$k = 6$$

$$T_7 = \binom{8}{6}x^0 = \frac{8 \cdot 7}{1 \cdot 2} = \boxed{28}$$