Ponavljanje za pismeni ispit – limes niza, geometrijski red

1. Izračunaj limese:

a. $\lim\limits_{n\to\infty} \dfrac{8n^2-5n+9}{1-3n-7n^2}$,

$$\lim_{n\to\infty} \frac{8n^2-5n+9\ /:n^2}{1-3n-7n^2\ /:n^2} = \lim_{n\to\infty} \frac{\frac{8n^2}{n^2}-\frac{5n}{n^2}+\frac{9}{n^2}}{\frac{1}{n^2}-\frac{3n}{n^2}-\frac{7n^2}{n^2}} = \lim_{n\to\infty} \frac{8-\frac{5}{n}+\frac{9}{n^2}}{\frac{1}{n^2}-\frac{3}{n}-7} = \boxed{-\frac{8}{7}}$$

b. $\lim\limits_{n\to\infty} \dfrac{\sqrt{n+1}+n}{\sqrt{n^2+1}+\sqrt{n}}$,

$$\lim_{n\to\infty} \frac{\sqrt{n+1}+n\ /:n}{\sqrt{n^2+1}+\sqrt{n}\ /:n} = \lim_{n\to\infty} \frac{\frac{\sqrt{n+1}}{n}+\frac{n}{n}}{\frac{\sqrt{n^2+1}}{n}+\frac{\sqrt{n}}{n}} = \lim_{n\to\infty} \frac{\sqrt{\frac{n+1}{n^2}}+1}{\sqrt{\frac{n^2+1}{n^2}}+\sqrt{\frac{n}{n^2}}} =$$

$$\lim_{n\to\infty} \frac{\sqrt{\frac{n}{n^2}+\frac{1}{n^2}}+1}{\sqrt{\frac{n^2}{n^2}+\frac{1}{n^2}}+\sqrt{\frac{1}{n}}} = \lim_{n\to\infty} \frac{\sqrt{\frac{1}{n}+\frac{1}{n^2}}+1}{\sqrt{1+\frac{1}{n^2}}+\sqrt{\frac{1}{n}}} = \frac{1}{\sqrt{1}} = \boxed{1}$$

c. $\lim\limits_{n\to\infty} \sqrt{n^2+n}-n$,

$$\lim_{n\to\infty} \sqrt{n^2+n}-n = \lim_{n\to\infty}\left(\sqrt{n^2+n}-n\right)\cdot\frac{\sqrt{n^2+n}+n}{\sqrt{n^2+n}+n} =$$

$$\lim_{n\to\infty} \frac{\left(\sqrt{n^2+n}-n\right)\left(\sqrt{n^2+n}+n\right)}{\sqrt{n^2+n}+n} = \lim_{n\to\infty} \frac{\left(\sqrt{n^2+n}\right)^2-n^2}{\sqrt{n^2+n}+n} = \lim_{n\to\infty} \frac{n^2+n-n^2}{\sqrt{n^2+n}+n} =$$

$$\lim_{n\to\infty} \frac{n\ /:n}{\sqrt{n^2+n}+n\ /:n} = \lim_{n\to\infty} \frac{\frac{n}{n}}{\frac{\sqrt{n^2+n}}{n}+\frac{n}{n}} = \lim_{n\to\infty} \frac{1}{\sqrt{\frac{n^2+n}{n^2}}+1} =$$

$$\lim_{n\to\infty} \frac{1}{\sqrt{\frac{n^2}{n^2}+\frac{n}{n^2}}+1} = \lim_{n\to\infty} \frac{1}{\sqrt{1+\frac{1}{n}}+1} = \frac{1}{\sqrt{1}+1} = \boxed{\frac{1}{2}}$$

d. $\lim\limits_{n\to\infty} \dfrac{3^n+4^n+5^n}{2^n+6^n}$.

$$\lim_{n\to\infty}\frac{3^n+4^n+5^n\ /:6^n}{2^n+6^n\ /:6^n}=\lim_{n\to\infty}\frac{\frac{3^n}{6^n}+\frac{4^n}{6^n}+\frac{5^n}{6^n}}{\frac{2^n}{6^n}+\frac{6^n}{6^n}}=\lim_{n\to\infty}\frac{\left(\frac{3}{6}\right)^n+\left(\frac{4}{6}\right)^n+\left(\frac{5}{6}\right)^n}{\left(\frac{2}{6}\right)^n+\left(\frac{6}{6}\right)^n}=$$

$$\lim_{n\to\infty}\frac{\left(\frac{1}{2}\right)^n+\left(\frac{2}{3}\right)^n+\left(\frac{5}{6}\right)^n}{\left(\frac{1}{3}\right)^n+1^n}=\frac{0}{1}=\boxed{0}$$

2. Odredi zbroj članova beskonačnog geometrijskog niza 2, 1.8, ...

$$a_1=2$$

$$q=\frac{1.8}{2}=\frac{\frac{18}{10}}{\frac{2}{1}}=\frac{18}{20}=\frac{9}{10}$$

$$S=\frac{a_1}{1-q}=\frac{2}{1-\frac{9}{10}}=\frac{2}{\frac{10-9}{10}}=\frac{\frac{2}{1}}{\frac{1}{10}}=\frac{20}{1}=\boxed{20}$$

3. Broj $0.2\dot{4}\dot{5}$ zapiši u obliku razlomka.

$$0.2\dot{4}\dot{5}=0.2+0.045+0.00045+0.0000045+\ldots=$$

$$\frac{2}{10}+\frac{45}{1000}+\frac{45}{100000}+\frac{45}{10000000}+\ldots=\frac{2}{10}+\frac{\frac{45}{1000}}{1-\frac{1}{100}}=\frac{2}{10}+\frac{\frac{45}{1000}}{\frac{100-1}{100}}=\frac{2}{10}+\frac{\frac{45}{1000}}{\frac{99}{100}}=$$

$$\frac{2}{10}+\frac{\frac{5}{10}}{\frac{11}{1}}=\frac{2}{10}+\frac{5}{110}=\frac{22+5}{110}=\boxed{\frac{27}{110}}$$

razlomci s brojnikom 45 čine geometrijski red u kojem je $a_1=\frac{45}{1000}$ i $q=\frac{1}{100}$