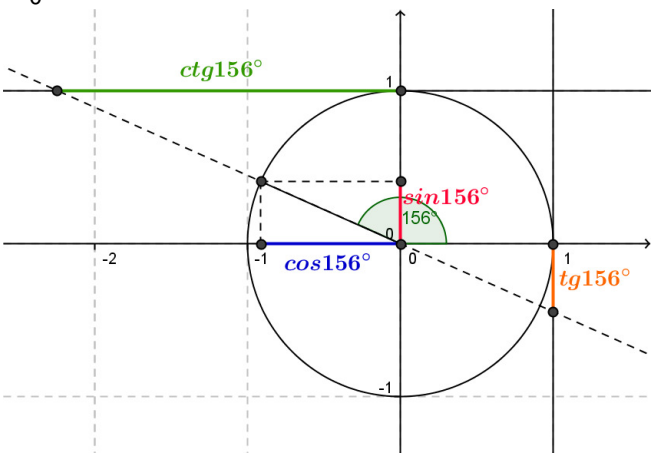
Ponavljanje za pismeni ispit – trigonometrijske funkcije realnog broja

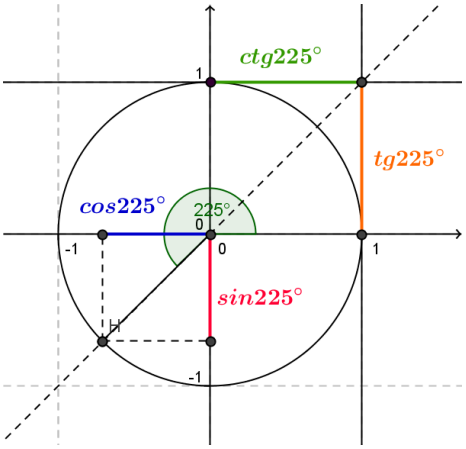
1. Odredi glavnu mjeru i na brojevnoj kružnici istakni vrijednosti svih trigonometrijskih funkcija (ako su definirane) kuta:
   a. $\alpha = 9876°$,

$\alpha = 9876° = 27 \cdot 360° + 156°$
$\alpha_0 = 156°$



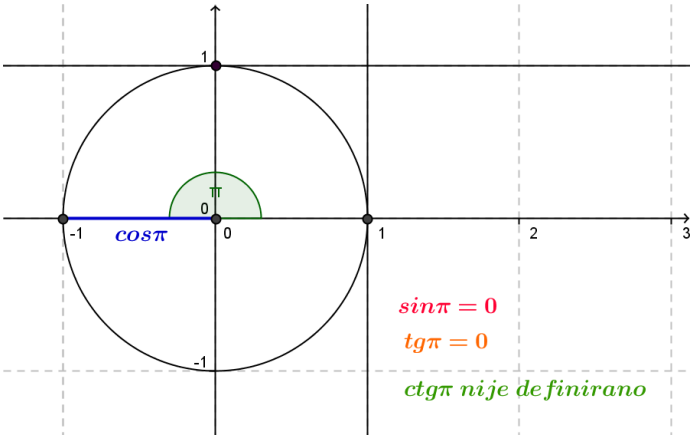
   b. $\alpha = -495°$,

$\alpha = -495° = -360° - 135° = -360° - 360° + 360° - 135° = -2 \cdot 360° + 225°$
$\alpha_0 = 225°$



   c. $t = -111\pi$,

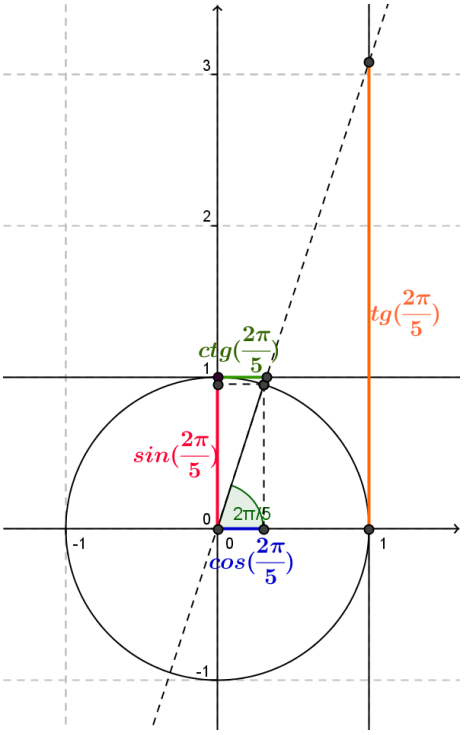
$t = -111\pi = -55 \cdot 2\pi - \pi = -55 \cdot 2\pi - 2\pi + 2\pi - \pi = -56 \cdot 2\pi + \pi$
$t_0 = \pi$



d. $t = \frac{872\pi}{5}$.

$$t = \frac{872\pi}{5} = 87 \cdot 2\pi + \frac{2\pi}{5}$$

$$t_0 = \frac{2\pi}{5}$$



2. Pretvori stupnjeve u radijane, a radijane u stupnjeve:

a. $1000°$,

$$1000° = 1000 \cdot \frac{\pi}{180}\,\text{rad} = \boxed{\frac{50\pi}{9}\,\text{rad}}$$

b. $71°13'47''$,

$$71°13'47'' = 71.23° = 71.23 \cdot \frac{\pi}{180}\,\text{rad} = \boxed{1.243\,\text{rad}}$$

c. $\frac{17\pi}{6}\,\text{rad}$,

$$\frac{17\pi}{6}\,\text{rad} = \frac{17\pi}{6} \cdot \frac{180°}{\pi} = \frac{17\pi}{6} \cdot \frac{180°}{\pi} = \frac{17\pi}{6} \cdot \frac{180°}{\pi} = \boxed{510°}$$

d. $-5.2\,\text{rad}$.

$$-5.2\,\text{rad} = -5.2 \cdot \frac{180°}{\pi} = -297{,}938° = \boxed{-297°56'16''}$$

3. Koristeći (ne)parnost i periodičnost trigonometrijskih funkcija, izračunaj:

a. $\sin(900°)$,

$$\sin(900°)=\sin(2\cdot 360°+180°)=\sin(180°)=\boxed{0}$$

b. $\cos(-1000\pi)$,

$$\cos(-1000\pi)=\cos(1000\pi)=\cos(500\cdot 2\pi+0)=\cos(0)=\boxed{1}$$

c. $\operatorname{tg}\left(\frac{35\pi}{2}\right)$,

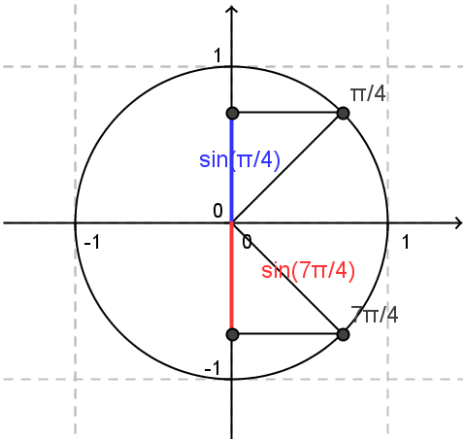
$$\operatorname{tg}\left(\frac{35\pi}{2}\right)=\operatorname{tg}\left(17\cdot\pi+\frac{\pi}{2}\right)=\operatorname{tg}\left(\frac{\pi}{2}\right)\ \boxed{\text{nije definirano}}$$

d. $\operatorname{ctg}\left(-\frac{13\pi}{2}\right)$,

$$\operatorname{ctg}\left(-\frac{13\pi}{2}\right)=-\operatorname{ctg}\left(\frac{13\pi}{2}\right)=-\operatorname{ctg}\left(6\cdot\pi+\frac{\pi}{2}\right)=-\operatorname{ctg}\left(\frac{\pi}{2}\right)=\boxed{0}$$

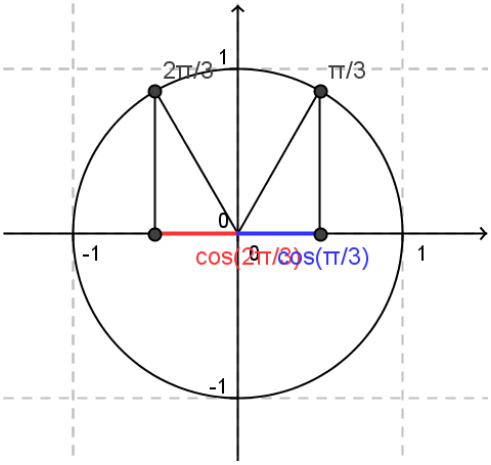
e. $\sin\left(-\frac{207\pi}{4}\right)$,

$$\sin\left(-\frac{207\pi}{4}\right)=-\sin\left(\frac{207\pi}{4}\right)=-\sin\left(25\cdot 2\pi+\frac{7\pi}{4}\right)=-\sin\left(\frac{7\pi}{4}\right)=-\left(-\frac{\sqrt{2}}{2}\right)=\boxed{\frac{\sqrt{2}}{2}}$$



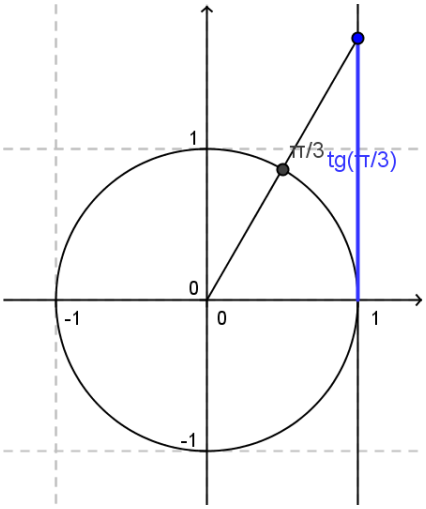
f. $\cos\left(\frac{332\pi}{3}\right)$,

$$\cos\left(\frac{332\pi}{3}\right)=\cos\left(55\cdot 2\pi+\frac{2\pi}{3}\right)=\cos\left(\frac{2\pi}{3}\right)=\boxed{-\frac{1}{2}}$$



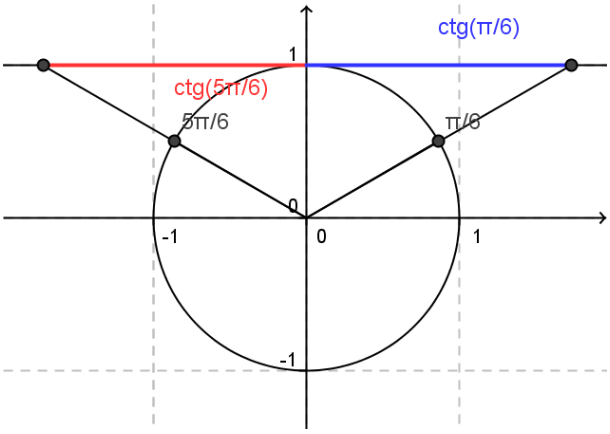
g. $\operatorname{tg}(-74040°)$,

$$\operatorname{tg}(-74040°) = -\operatorname{tg}(74040°) = -\operatorname{tg}(411 \cdot 180° + 60°) = -\operatorname{tg}(60°) = \boxed{-\sqrt{3}}$$



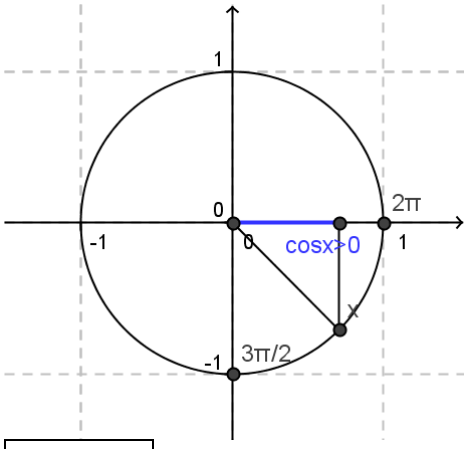
h. $\operatorname{ctg}\left(\frac{53\pi}{6}\right)$.

$$\operatorname{ctg}\left(\frac{53\pi}{6}\right) = \operatorname{ctg}\left(8 \cdot \pi + \frac{5\pi}{6}\right) = \operatorname{ctg}\left(\frac{5\pi}{6}\right) = \boxed{-\sqrt{3}}$$



4. Ako je $\sin x = -\frac{21}{29}$ i $x \in \left\langle \frac{3\pi}{2}, 2\pi \right\rangle$, odredi vrijednosti ostalih trigonometrijskih funkcija broja x.

$$\sin^2 x + \cos^2 x = 1$$
$$\left(-\frac{21}{29}\right)^2 + \cos^2 x = 1$$
$$\frac{441}{841} + \cos^2 x = 1$$
$$\cos^2 x = 1 - \frac{441}{841}$$
$$\cos^2 x = \frac{841 - 441}{841}$$
$$\cos^2 x = \frac{400}{841} \quad / \sqrt{\ }$$
$$\cos x = \pm\frac{20}{29}$$



$$\boxed{\cos x = \frac{20}{29}}$$

$$tgx = \frac{\sin x}{\cos x}$$
$$tgx = \frac{-\frac{21}{29}}{\frac{20}{29}}$$
$$tgx = \frac{-\frac{21}{1}}{\frac{20}{1}}$$
$$\boxed{tgx = -\frac{21}{20}}$$

$$ctgx = \frac{1}{tgx}$$
$$\boxed{ctgx = -\frac{20}{21}}$$

5. Ako je $\operatorname{ctgt} = \frac{\sqrt{3}}{3}$ i $t \in \left\langle 8\pi, \frac{17\pi}{2} \right\rangle$, odredi vrijednosti ostalih trigonometrijskih funkcija broja x.

$$\operatorname{tgt} = \frac{1}{\operatorname{ctgt}}$$

$$\operatorname{tgt} = \frac{3}{\sqrt{3}} = \frac{3}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{3\sqrt{3}}{3} = \sqrt{3}$$

$$\boxed{\operatorname{tgt} = \sqrt{3}}$$

$$\operatorname{ctg}^2 t + 1 = \frac{1}{\sin^2 t}$$

$$\left(\frac{\sqrt{3}}{3}\right)^2 + 1 = \frac{1}{\sin^2 t}$$

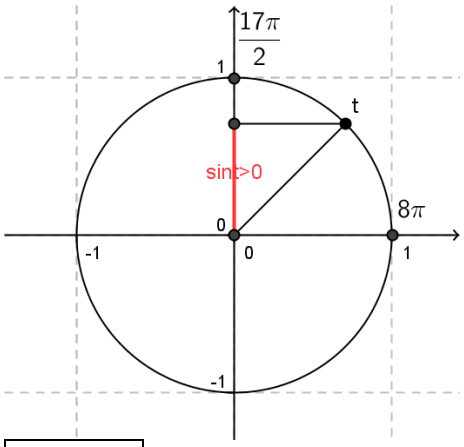
$$\frac{1}{3} + 1 = \frac{1}{\sin^2 t}$$

$$\frac{1+3}{3} = \frac{1}{\sin^2 t}$$

$$\frac{4}{3} = \frac{1}{\sin^2 t}$$

$$\sin^2 t = \frac{3}{4} \quad / \sqrt{\ }$$

$$\sin t = \pm \frac{\sqrt{3}}{2}$$



$$\boxed{\sin t = \frac{\sqrt{3}}{2}}$$

$$\operatorname{tgt} = \frac{\sin t}{\cos t}$$

$$\cos t = \frac{\sin t}{\operatorname{tgt}}$$

$$\cos t = \frac{\frac{\sqrt{3}}{2}}{\frac{\sqrt{3}}{1}}$$

$$\boxed{\cos t = \frac{1}{2}}$$