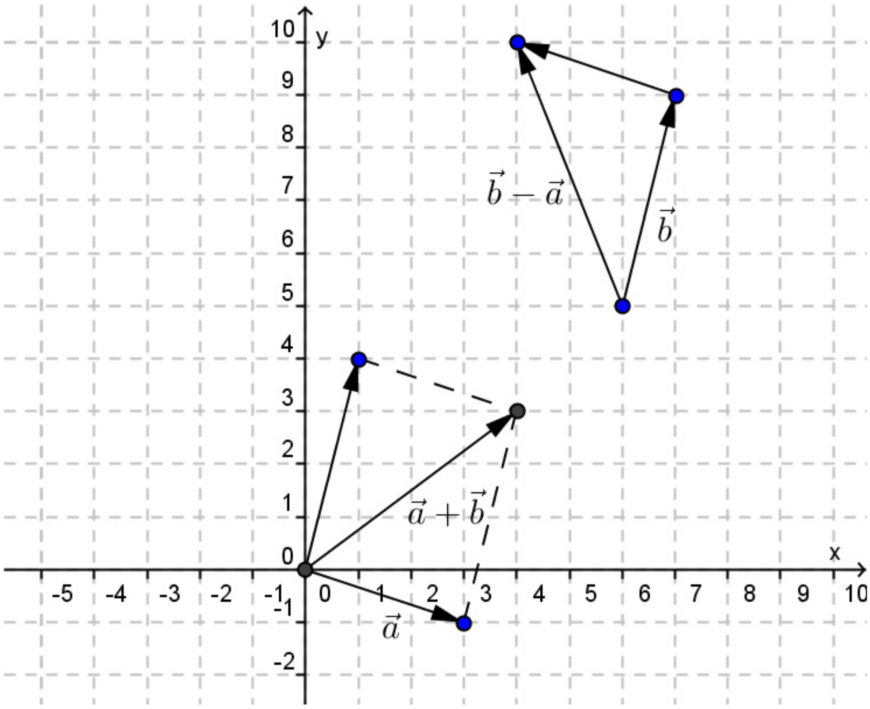
Ponavljanje za pismeni ispit – vektori

1. Zadani su vektori $\vec{a} = 3\vec{i} - \vec{j}$ i $\vec{b} = \vec{i} + 4\vec{j}$. U koordinatnom sustavu nacrtaj:
   a. vektor $\vec{a}$ s početkom u ishodištu koordinatnog sustava,
   b. vektor $\vec{b}$ s početkom u točki (6,5),
   c. vektor $\vec{a} + \vec{b}$ (pravilom paralelograma),
   d. vektor $\vec{b} - \vec{a}$ (pravilom trokuta).



2. Vektor $\vec{c} = -\vec{m} + 7\vec{n}$ prikaži kao linearnu kombinaciju vektora $\vec{a} = 3\vec{m} - \vec{n}$ i $\vec{b} = \vec{m} - 2\vec{n}$.

$\vec{c} = \alpha\vec{a} + \beta\vec{b}$

$-\vec{m} + 7\vec{n} = \alpha\left(3\vec{m} - \vec{n}\right) + \beta\left(\vec{m} - 2\vec{n}\right)$

$-\vec{m} + 7\vec{n} = 3\alpha\vec{m} - \alpha\vec{n} + \beta\vec{m} - 2\beta\vec{n}$

$-\vec{m} + 7\vec{n} = (3\alpha + \beta)\vec{m} + (-\alpha - 2\beta)\vec{n}$

$3\alpha + \beta = -1 \ /\cdot 2$

$\underline{-\alpha - 2\beta = 7}$

$6\alpha + 2\beta = -2$

$\underline{-\alpha - 2\beta = 7}$

$5\alpha = 5 \ / : 5$

$\alpha = 1$

$-1 - 2\beta = 7$

$-2\beta = 7 + 1$

$-2\beta = 8 \ / : (-2)$

$\beta = -4$

$$\vec{c} = \alpha\,\vec{a} + \beta\,\vec{b}$$

$$\boxed{\vec{c} = \vec{a} - 4\,\vec{b}}$$

3. Zadane su točke A(1,4), B(3,3), C(-5,3) i D(1,5). Odredi vektor $5\,\overrightarrow{AB} - \frac{1}{2}\overrightarrow{CD} + \overrightarrow{DB}$.

$$\overrightarrow{AB} = (3-1)\,\vec{i} + (3-4)\,\vec{j} = 2\,\vec{i} - \vec{j}$$

$$\overrightarrow{CD} = (1+5)\,\vec{i} + (5-3)\,\vec{j} = 6\,\vec{i} + 2\,\vec{j}$$

$$\overrightarrow{DB} = (3-1)\,\vec{i} + (3-5)\,\vec{j} = 2\,\vec{i} - 2\,\vec{j}$$

$$5\,\overrightarrow{AB} - \frac{1}{2}\overrightarrow{CD} + \overrightarrow{DB} = 5\left(2\,\vec{i} - \vec{j}\right) - \frac{1}{2}\left(6\,\vec{i} + 2\,\vec{j}\right) + \left(2\,\vec{i} - 2\,\vec{j}\right) = 10\,\vec{i} - 5\,\vec{j} - 3\,\vec{i} - \vec{j} + 2\,\vec{i} - 2\,\vec{j}$$

$$\boxed{5\,\overrightarrow{AB} - \frac{1}{2}\overrightarrow{CD} + \overrightarrow{DB} = 9\,\vec{i} - 8\,\vec{j}}$$

4. Odredi vektor duljine 8 i suprotne orijentacije od vektora $\vec{a} = -12\,\vec{i} + 5\,\vec{j}$.

$$|\vec{a}| = \sqrt{(-12)^2 + 5^2} = \sqrt{144 + 25} = \sqrt{169} = 13$$

$$\vec{a}_0 = \frac{1}{|\vec{a}|} \cdot \vec{a} = \frac{1}{13}\left(-12\,\vec{i} + 5\,\vec{j}\right)$$

Traženi vektor je $\boxed{-8\vec{a}_0 = -\frac{8}{13}\left(-12\,\vec{i} + 5\,\vec{j}\right) = \frac{96}{13}\,\vec{i} - \frac{40}{13}\,\vec{j}}$

5. Odredi kut između vektora $\overrightarrow{AB}$ i $\overrightarrow{CD}$, ako su zadane točke A(1,4), B(3,3), C(-5,3) i D(1,5).

$$\overrightarrow{AB} = (3-1)\,\vec{i} + (3-4)\,\vec{j} = 2\,\vec{i} - \vec{j}$$

$$\overrightarrow{CD} = (1+5)\,\vec{i} + (5-3)\,\vec{j} = 6\,\vec{i} + 2\,\vec{j}$$

$$\cos\angle(\overrightarrow{AB}, \overrightarrow{CD}) = \frac{\overrightarrow{AB} \cdot \overrightarrow{CD}}{|\overrightarrow{AB}| \cdot |\overrightarrow{CD}|} = \frac{2 \cdot 6 - 1 \cdot 2}{\sqrt{2^2 + (-1)^2} \cdot \sqrt{6^2 + 2^2}} = \frac{12 - 2}{\sqrt{4+1} \cdot \sqrt{36+4}} = \frac{10}{\sqrt{5} \cdot \sqrt{40}} =$$

$$= \frac{10}{\sqrt{200}} = \frac{10}{\sqrt{100 \cdot 2}} = \frac{10}{10\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\boxed{\angle(\overrightarrow{AB}, \overrightarrow{CD}) = \cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = 45^\circ}$$

6. Ako za vektore $\vec{a}$ i $\vec{b}$ vrijedi da je $|\vec{a}| = 4$, $|\vec{b}| = 5$ i $\angle(\vec{a}, \vec{b}) = \frac{3\pi}{4}$, odredi $|2\vec{a} - \vec{b}|$.

$$|2\vec{a} - \vec{b}|^2 = \left(2\vec{a} - \vec{b}\right)^2 = 4(\vec{a})^2 - 4\vec{a}\cdot\vec{b} + (\vec{b})^2 = 4|\vec{a}|^2 - 4|\vec{a}||\vec{b}|\cos\angle(\vec{a}, \vec{b}) + |\vec{b}|^2 =$$

$$4\cdot 4^2 - 4\cdot 4\cdot 5\cdot \cos\frac{3\pi}{4} + 5^2 = 64 - 80\cdot\left(-\frac{\sqrt{2}}{2}\right) + 25 = 89 + 40\sqrt{2}$$

$$|2\vec{a} - \vec{b}|^2 = 89 + 40\sqrt{2} \approx 145.569 \quad /\sqrt{\ }$$

$$\boxed{|2\vec{a} - \vec{b}| \approx 12.065}$$