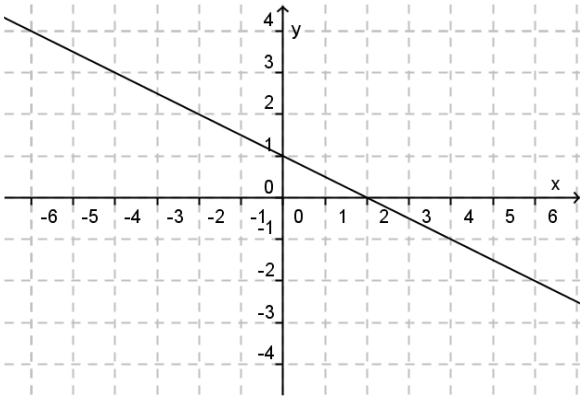
Ponavljanje za pismeni ispit – koordinatni sustav, pravac

1. Zadana je afina funkcija $f(x) = -\frac{1}{2}x + 1$.
   a. Skiciraj graf funkcije.



   b. Odredi nultočku funkcije.

$a = -\frac{1}{2}$, $b = 1$

$x_0 = -\frac{b}{a}$

$x_0 = -\frac{1}{-\frac{1}{2}}$

$\boxed{x_0 = 2}$ - nultočka

   c. Odredi točke u kojima graf siječe koordinatne osi.

$\boxed{(x_0, 0) = (2, 0)}$ - presjek s x-osi

$\boxed{(0, b) = (0, 1)}$ - presjek s y-osi

   d. Odredi površinu trokuta kojeg graf zatvara s koordinatnim osima.

$P = \frac{|2 \cdot 1|}{2}$

$\boxed{P = 1}$ - površina trokuta

   e. Odredi za koje je vrijednosti x ispunjena nejednakost $f(x) \geq 2$.

$f(x) = -\frac{1}{2}x + 1$

$f(x) \geq 2$

$-\frac{1}{2}x + 1 \geq 2$

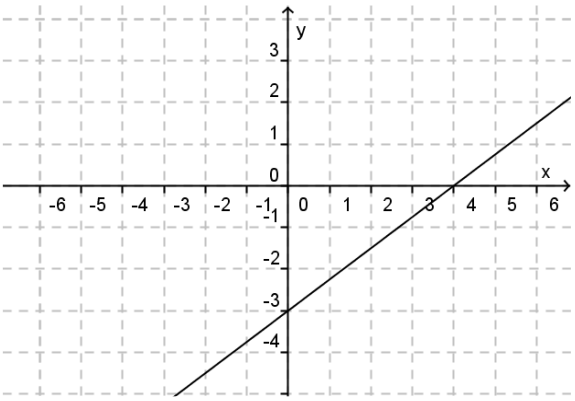
$-\frac{1}{2}x \geq 2 - 1$

$-\frac{1}{2}x \geq 1 \,/\cdot(-2)$

$x \leq -2$

$\boxed{x \in \langle -\infty, -2]}$

2. Na slici je prikazan pravac.



a. Odredi eksplicitni, implicitni i segmentni oblik jednadžbe pravca.

sa slike se vidi da je odsječak pravca na x-osi jednak 4, a na y-osi jednak -3

$\boxed{\frac{x}{4}+\frac{y}{-3}=1}$ - segmentni oblik jednadžbe

$\frac{x}{4}+\frac{y}{-3}=1 / \cdot 12$

$3x-4y=12$

$\boxed{3x-4y-12=0}$ - implicitni oblik jednadžbe

$-4y=-3x+12 / :(-4)$

$\boxed{y=\frac{3}{4}x-3}$ - eksplicitni oblik jednadžbe

b. Odredi jednadžbu pravca usporednog (paralelnog) sa zadanim, kroz ishodište koordinatnog sustava.

$a=\frac{3}{4}$, T(0,0)

$y-y_1=a(x-x_1)$

$y-0=\frac{3}{4}(x-0)$

$\boxed{y=\frac{3}{4}x}$

c. Odredi jednadžbu pravca okomitog na zadani, kroz točku T(-1,2).

$a=-\frac{4}{3}$, T(-1,2)

$y-y_1=a(x-x_1)$

$y-2=-\frac{4}{3}(x+1)$

$y-2=-\frac{4}{3}x-\frac{4}{3}$

$y=-\frac{4}{3}x-\frac{4}{3}+2$

$y=-\frac{4}{3}x+\frac{-4+6}{3}$

$\boxed{y=-\frac{4}{3}x+\frac{2}{3}}$

3. Afina funkcija zadana je tablicom.

x	1	2	3
f(x)	1	4	7

a. Odredi funkciju.

uzmemo bilo koje dvije točke iz tablice
(1,1), (2,4)

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 1 = \frac{4-1}{2-1}(x-1)$$

$$y - 1 = 3(x-1)$$

$$y - 1 = 3x - 3$$

$$y = 3x - 3 + 1$$

$$y = 3x - 2$$

$$\boxed{f(x) = 3x - 2}$$

b. Koju vrijednost poprima ta funkcija za x=8?

$$f(x) = 3x - 2$$

$$f(8) = 3 \cdot 8 - 2$$

$$\boxed{f(8) = 22}$$

c. Nalazi li se točka (5,-3) na grafu te funkcije?

$$y = 3x - 2$$

(5,-3)

$$-3 = 3 \cdot 5 - 2$$

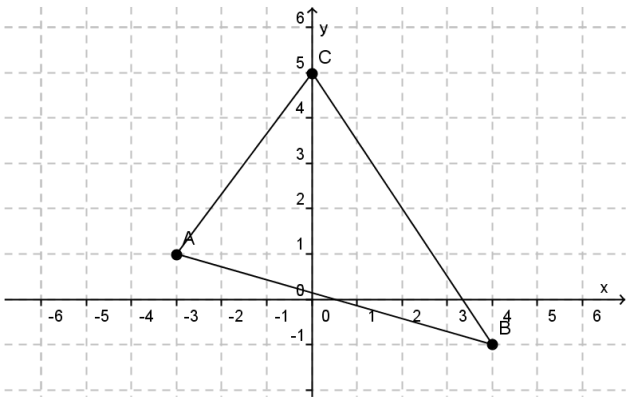
$$-3 = 15 - 2$$

$$-3 = 13$$

Koordinate točke ne zadovoljavaju jednadžbu pravca.

$\boxed{\text{Ne.}}$

4. Na slici je prikazan trokut ABC.



a. Odredi opseg trokuta.

A(-3,1), B(4,-1), C(0,5)

$$a = |BC| = \sqrt{(0-4)^2 + (5+1)^2} = \sqrt{(-4)^2 + 6^2} = \sqrt{16+36} = \sqrt{52}$$

$$b = |AC| = \sqrt{(0+3)^2 + (5-1)^2} = \sqrt{3^2 + 4^2} = \sqrt{9+16} = \sqrt{25} = 5$$

$$c = |AB| = \sqrt{(4+3)^2 + (-1-1)^2} = \sqrt{7^2 + (-2)^2} = \sqrt{49+4} = \sqrt{53}$$

$$O = a + b + c$$

$$\boxed{O = \sqrt{52} + 5 + \sqrt{53}}$$

b. Odredi površinu trokuta.

A(-3,1), B(4,-1), C(0,5)

$$P = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

$$P = \frac{1}{2}\left|-3(-1-5) + 4(5-1) + 0(1+1)\right|$$

$$P = \frac{1}{2}\left|-3(-6) + 4 \cdot 4\right|$$

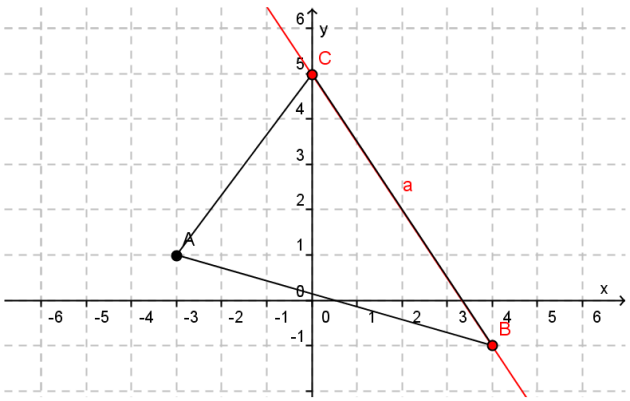
$$P = \frac{1}{2}\left|18 + 16\right|$$

$$P = \frac{1}{2}\left|34\right|$$

$$P = \frac{1}{2} \cdot 34$$

$$\boxed{P = 17}$$

c. Odredi jednadžbu pravca na kojem leži stranica a trokuta.



B(4,-1), C(0,5)

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y + 1 = \frac{5+1}{0-4}(x-4)$$

$$y + 1 = \frac{6}{-4}(x-4)$$

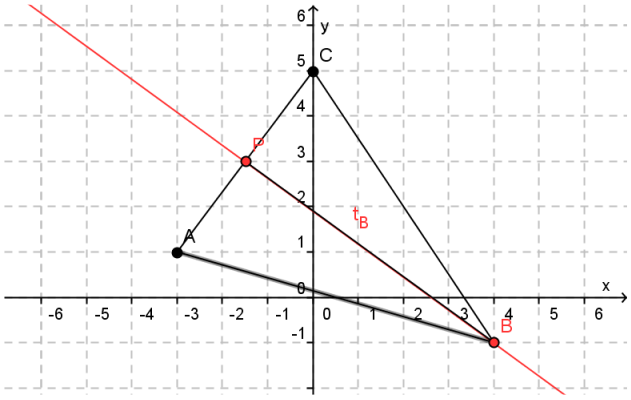
$$y + 1 = -\frac{3}{2}(x-4)$$

$$y + 1 = -\frac{3}{2}x + \frac{12}{2}$$

$$y = -\frac{3}{2}x + 6 - 1$$

a ... $\boxed{y = -\frac{3}{2}x + 5}$

d. Odredi jednadžbu pravca na kojem leži težišnica iz vrha B trokuta.



A(-3,1), C(0,5)

$$x_P = \frac{x_A + x_C}{2} = \frac{-3+0}{2} = -\frac{3}{2}$$

$$y_P = \frac{y_A + y_C}{2} = \frac{1+5}{2} = \frac{6}{2} = 3$$

B(4,-1), $P\left(-\frac{3}{2},3\right)$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y + 1 = \frac{3+1}{-\frac{3}{2} - 4}(x-4)$$

$$y + 1 = \frac{4}{\frac{-3-8}{2}}(x-4)$$

$$y + 1 = \frac{\frac{4}{1}}{\frac{-11}{2}}(x-4)$$

$$y + 1 = -\frac{8}{11}(x-4)$$

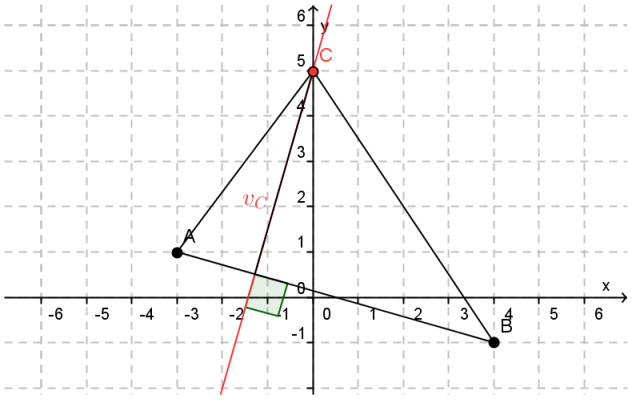
$$y + 1 = -\frac{8}{11}x + \frac{32}{11}$$

$$y = -\frac{8}{11}x + \frac{32}{11} - 1$$

$$y = -\frac{8}{11}x + \frac{32-11}{11}$$

t_B ... $\boxed{y = -\frac{8}{11}x + \frac{21}{11}}$

e. Odredi jednadžbu pravca na kojem leži visina iz vrha C trokuta.



A(-3,1), B(4,-1)

$$a_{AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-1}{4+3} = -\frac{2}{7}$$

$$a_{v_c} = \frac{7}{2}, \text{ C(0,5)}$$

$$y - y_1 = a(x - x_1)$$

$$y - 5 = \frac{7}{2}(x - 0)$$

$$v_c \ldots \boxed{y = \frac{7}{2}x + 5}$$